THE SOUTHERN CONFEDERACY ARITHMETIC,

FOR

Common Schools and Academies,

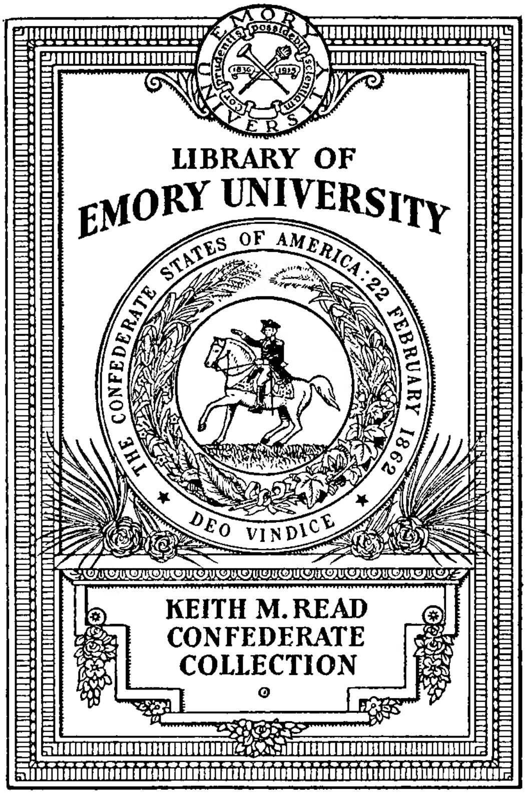
WITH A

PRACTICAL SYSTEM OF BOOK-KEEPING

BY SINGLE ENTRY.

BY REV. CHARLES E. LEVERETT, A.M.,
AUTHOR OF THE SOUTHERN CONFEDERACY CLASS READERS,
ETC., ETC.

AUGUSTA, GA.:
PRINTED AND PUBLISHED BY J. T. PATERSON & CO.
1864.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Entered according to Act of Congress in the year 1864, by

J. T. PATERSON & CO.,

in the Clerk's Office of the District Court of the Confederate States for the Northern District of Georgia.

PREFACE.

A FEW years since, when, for reasons unnecessary to be repeated here, it was thought advisable that the South should have the preparation of her own educational literature, the author was asked to aid in the enterprise conducting to that end. At first, as not being exactly in the line of his professional service, he declined; but, after due deliberation and consultation, he assented, and his earliest contribution is the volume which this preface introduces. In offering this he begs to say, that while a long scholastic experience has given him peculiar facilities for his work, his deep interest in the proper training of the young has materially aided in what, he trusts, will tend to a good result. Nor does he now hesitate to affirm that it will be a source of the deepest gratification to him, if, through his humble agency, any text book of the section, which has developed the most malignant principles and exhibited the cruelest practice, should be forever excluded from a Southern school.

In the preparation of this volume, the author has been guided by what he understood had long been needed—a simple and comprehensive work—one that could be studied both with profit and pleasure. He hopes to have reached, at least partly, what seemed to him desirable. The books, and the most popular ones, which the Northern market has in immense numbers exported to the South, have very glaring defects, especially in the attempt to teach too little, and again too much. The

consequence has been embarrassment to the common order of minds, and a tendency to make a most pleasing science an occult study, and a useful and necessary branch of science a distasteful task. In this way, that course, without which no education is complete, either in its relation to the sciences connected with the reasoning powers, or in its practical addresses in commercial engagements, and in the daily applications of domestic finance, is made to sustain a most depreciated stand.

It has been the author's aim to avoid prolixity. To acquire numerical skill, a pupil should not become weary of his work. Arithmetics, generally, because too long, are tiresome, and the end—which the scholar always has in view—appears—

"The weary wild, that lengthens as we go."

This book is purposely short, and, as for the most part, the secret of scholarly success is the culture from the review—the nail to fasten science to the mind—it should be repeatedly used before the higher volume of the series is taken up.

It has been an additional aim to give, at each progressive step, clear explanations. In these, when it could be done, the rule for operations is introduced, and by this plan has been avoided what to young pupils is a very mystifying and objectionable part of Arithmetical study—the dry formula of directions. In regard to the explanations, much must be left to the teacher, and he is an unintelligent one who does not make plain, by illustrations, certain truths, which no Arithmetic, unless it be expanded to an unreasonable length, could communicate. In this connection it may be well to say that it would have been perfectly easy to have introduced a very frequent analysis, and with it have blended questions, after

the manner of some Arithmeticians, in order to combine the mental and written departments; but, after trying all forms, his experience has held it preferable to keep them separate. When too many things are presented together there is confusion, and the contemplated object is not secured. For the encouragement of the pupil, even if his be not a mind, strictly speaking, mathematically framed, it may be said, if he have well studied the Intellectual volume of this series, he can hardly fail, by the time he has completed this, with its review, to comprehend to a fair degree what Arithmetic has been defined, the art of numbers, and the science or knowledge of computation.

The marginal notes, it is thought, will prove highly useful and acceptable as a ready index of subjects and a form for questions, without the formality of their shape. This new feature has received from intelligent teachers very gratifying approbation. The divisions into Introductory, Higher, Fractional, Proportional, Commercial, Radical and Miscellaneous Arithmetic, it is hoped, will be a help both to the scholar and instructor. For the convenience of the latter, a key, containing answers, will accompany the volume.

COLUMBIA, S. C., *Feb.*, 1864.

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BOOK-KEEPING BY SINGLE ENTRY.

COMMON ARITHMETIC.

SECTION I.

SIMPLE NUMBERS.

Arithmetic defined

ARTICLE **1.** Arithmetic—a Greek derivative—signifies simply the art of numbers. It is now understood to comprehend in its expression the science or knowledge of computation.

What a number is.

2. A number is what is used to describe a quantity, and is either a unit, as the number *one*; or a collection of units, as *two*, *ten*, *one hundred*.

Numbers simple and compound.

3. A number is either *simple* or *compound*. It is simple when it expresses a single collection of things, as *five trees*: compound, a collection of varieties, as *five trees and six apples*.

Arithmetical science and art explained.

4. Arithmetical science shows the *values and connections* of numbers: art, their *structure*, either in complex form, or common relation.

An operation: a result.

5. An arithmetical operation is *work done* by the employment of numbers; the result of the work is *the answer*.

A rule: a sum.

6. A rule is *a direction* to a result; and a *sum*, the proposed problem for the exercise of a rule.

A sum analyzed.

7. The analysis of a sum is *its separation* into component parts.

The number of the arithmetical basis, and extent.

8. There are *six different names* in arithmetical use, and what they express is more or less introduced *into all operations not simply elementary*: these are,

Arithmetical basis—names.

9. *Notation*, *Numeration*, *Addition*, *Subtraction*, *Multiplication and Division.*

Notation defined.

10. *Notation*—the simplest expression of a number—shows how to read or write an arithmetical proposition.

Three forms of notation.

11. The form in which an arithmetical proposition is given, is in *letters*, *figures*, or *words*: the first is known as *Roman* notation; the second, *Arabic*; the third, *Verbal*, or, as when we write *five*, *fifty*, *five hundred.*

Roman notation.

12. *The Roman Notation* employs seven capital letters: thus I, expresses *one*: V, *five*: X, *ten*; L, *fifty*; C, *one hundred*; D, *five hundred*, and M, *one thousand*. From these we have the following

13. TABLE OF ROMAN NUMERALS.

Roman numerals.

I.	1.	XXIV.	24.
II.	2.	XXV.	25.
III.	3.	XXIX.	29.
IV.	4.	XXX.	30.
V.	5.	XL.	40.
VI.	6.	L.	50.
VII.	7.	LXX.	70.
VIII.	8.	XC.	90.
IX.	9.	C.	100.
X.	10.	CCC.	300.
XI.	11.	D.	500.
XII.	12.	DCCCC.	900.
XIII.	13.	M.	1000.
XIV	14.	MD.	1500.
XV.	15.	MDC.	1600.
XVI.	16.	MDCLXV.	1665.
XVII.	17.	MDCCXLIX.	1749.
XVIII.	18.	MDCCCXVI.	1816.
XIX.	19.	MDCCCLVII.	1857.
XX.	20.	MDCCCLXII.	1862.
XXII.	22.	MM.	2000.

General expressions of Roman numerals.

14. When two or more equivalent letters are united, or when one of less value follows one of greater, we have an expression of their connected worth; thus, XX equals 20; XLV, 45; but when a letter not equivalent comes between two of greater value, it is to be *subtracted*, or taken from that which follows; while its remainder must be *added* to that which preceded it; thus, XIV equals 14; CXL, 140.

Certain expressions explained.

15. Sometimes a letter of lower value stands at the left of one of higher; then the difference of their worth is expressed; thus IX, or one from ten, equals 9; XL, or ten from fifty equals 40.

Exercises in Roman notation.

16. EXERCISES IN ROMAN NOTATION.

Six,	VI.
Eight,	
Eighteen,	
Twenty-eight,	
Thirty,	
Forty-nine,	XLIX.
Eighty-four,	
One hundred and ten,	
Five hundred and thirty-three,	

Six hundred and nine, DCIX.
Seven hundred and fourteen,
One thousand and nineteen,
One thousand eight hundred and sixty-two.

17. *Arabic Notation* is the method to express numbers by *figures*. *Ten* are used, but the first, 0, is called indiscriminately a *zero*, *cipher*, or *naught*. When by itself it signifies *nothing*; but when *combined with other figures*, it is in name alone a cipher. The figures, exclusive of the 0, are, Arabic notation. The cipher, when without and with value.

18. 1, 2, 3, 4, 5, 6, 7, 8, 9,
one, two, three, four, five, six, seven, eight, nine,
and each represents the number written beneath, as just given. Figures and their corresponding numbers.

19. There is no single figure to express the number ten. Hence we are obliged to place an 0 at the right hand of 1; thus, 10 *is ten*. The manner of forming ten in figures.

20. The value of a figure is increased *ten-fold* by removing it *one place towards the left*; *a hundred-fold* by removing it *two places*; *a thousand*, by removing it *three*, and so on. How values are increased.

21. The figures of large numbers are more easily read when separated by commas *into periods of three*. Separation of numbers.

22. By the method in common use, *the first, or right-hand period*, contains *units*, *tens and hundreds*, and is known as *the period of unity*; *the second* contains *thousands*, *tens of thousands, and hundreds of thousands*, and is known as *the period of thousands*; and *the third* contains *millions, tens of millions, and hundreds of millions*: and so on, according to the following Names and contents of the separating periods.

23. NUMERATION TABLE. The numeration table.

Tens of Trillions,	Trillions,	Hundreds of Billions,	Tens of Billions,	Billions,	Hundreds of Millions,	Tens of Millions,	Millions,	Hundreds of Thousands,	Tens of Thousands,	Thousands,	Hundreds,	Tens,	Units,
5	9,	6	9	5,	5	6	3,	8	0	1.	0	1	0
5th period. Trillions.		4th period. Billions.			3d period. Millions.			2d period. Thousands.			1st period. Units.		

Value of the given table.

24. Expressed in words, the value of the figures in the above table, is fifty-nine trillions, six hundred and ninety-five billions, five hundred and sixty-three millions, eight hundred and one thousand, and ten.

The table extended if necessary.

25. By the adoption of a name for further periods, this table can be indefinitely extended. For common purposes, it is sufficiently large.

Numeration defined.

26. *Numeration* is simply the art of reading numbers that have been expressed by figures.

Figures of each period.

27. Each period always contains *three figures*, except the *last*, which may have *one figure*, or *two or three figures*. In the table given, it will be seen, that in the fifth period, no hundreds are given.

To read numbers.

28. To read numbers:

I. Separate, by a comma, the number into periods of three figures each, beginning at the right hand.

II. Name the order of each figure, beginning at the right hand; as units, tens, hundreds, and so on, to the extent required.

III. Then commence at the left hand, and read each period as if it stood alone.

EXERCISES IN NUMERATION.

29. Let the pupil separate by commas the following numbers, viz:

Exercises in numeration.

1.	48	11.	468765201
2.	127	12.	32575654321
3.	5654	13.	718950
4.	9705	14.	985674112
5.	32693	15.	6785403
6.	57865	16.	25687145
7.	69432	17.	112543764
8.	234356	18.	22789532140
9.	785643	19.	3642005428
10.	305400	20.	5001400

To write numbers.

30. To write numbers:

I. Commence at the left hand, and write each period in order.

II. When a full vacant period occurs, ciphers must occupy the space.

NOTE.—With a little care, the pupil will be able to write any number. Let him remember that each period is to be expressed. When no number is given, a cipher must be employed. No unit, ten, or hundred must be left without expression.

EXAMPLES.

31. Write the following in figures : Examples for practice in numeration

1. Twenty-five,
2. Thirty, 30
3. Seventy-eight,
4. Four hundred,
5. Six hundred and two, 602
6. Nine hundred and sixty-five,
7. One thousand and one,
8. Fifty-three hundred and fifty-six,
9. One hundred thousand, 100,000
10. One hundred thousand and eighty,
11. Six hundred thousand and two,
12. One million, three hundred and twenty-one thousand,
13. Two hundred and twenty-five billions, four hundred and sixty-three millions, seven hundred and ninety-eight thousand, two hundred and thirty-four.
14. Three billions and sixty-five, 3,000,000,065
15. Nine billions, two millions, twenty-five thousand, one hundred,
16. Forty billions, one hundred and twenty-seven millions, and ninety-nine.
17. Three trillions, sixty-one thousand, and seven.
18. One hundred and twenty billions, one thousand, and one hundred.
19. Two billions, twenty-two millions, two hundred and twenty-two thousand, two hundred and twenty-two, 2,022,222,222
20. One trillion, one hundred thousand.

NOTATION AND NUMERATION.

1. The earliest method of denoting numbers was probably that of representing each unit by a separate sign. No perfectly convenient method was found, until the Arabic figures, or Digits, and the present Decimal system were employed. These figures, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, denote nothing, one unit, two units, three units, and so on. The earliest method of denoting numbers. What the figures in use denote.

2. To express numbers in excess of 9, availment is made of the law that assigns higher values to figures, according to their position. According to this law, any figure at the To express a value in excess of 9.

A law in figures. The result of the position.

left of another figure expresses ten times the value that it would express if it occupied the place of the figure at its right. Consequently, a higher order of units arises in succession.

Different values expressed by the figure 1. The zero—its use.

3. In illustration of this law, let the different values expressed by the figure 1 be noticed. Standing by itself, or at the right hand of other figures, 1 represents 1 unit of the first order; in the second place towards the left, thus 10, it represents 1 ten, which is one unit of the second degree; when in the third place, thus, 100, it represents 1 hundred; which is one unit of the third order, and so on. The ciphers here employed are without value in themselves. They are only used to occupy a place.

Unity of the second order. Third order. Intermediate numbers. A more consistent nomenclature.

4. The units of the second order, or the tens, are successively named 10, ten, 20, twenty, 30, thirty, and so on to 90, ninety. The units of the third order, or the hundreds, are named 100, one hundred, 200, two hundred, 300, three hundred, and so on to 900, nine hundred. The numbers between 10 and 20 are named 11, eleven, 12, twelve, 13, thirteen, and so on to 19, nineteen. The intermediate numbers in other tens are similarly denoted, but their designation is taken from the names of their respective units; thus, 21, twenty-one, 22, twenty-two, 23, twenty-three, and so on; 31, thirty-one, 32, thirty-two, &c.; 41, forty-one, &c. Compound names applied to the numbers between 10 and 20, might be more consistent, thus, 11, ten-one, 12, ten-two, 13, ten-three, &c.; but those in common use are sufficiently intelligible.

The numeration table.

5. As the first three places of figures are appropriated to simple units, tens and hundreds, so every succeeding three places are appropriated to the units, tens and hundreds of higher denominations. Adopting a name for every three degree of units, the table can be indefinitely extended.

The table extended.

Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
233,	104,	395,	473,	258,	333,	573,	820,	765,	569,	321,	560.

Write in figures

1. Three octillions, four quintillions, two millions and twenty-nine.

2. Twenty-five septillions, three hundred and thirty-five quadrillions, thirty-seven billions, two thousand and five.

3. Ten decillions, eight sextillions, one thousand and one hundred.
4. Three hundred quadrillions, four billions and sixty-five.
5. Sixty-five octillions, thirty-five trillions and ninety-five.
6. Four sextillions, two thousand and three.

ADDITION.

32. *Addition* is the putting together of one or more numbers, to find their amount. Addition defined.

33. The sign + is called *plus*, and signifies *more*. When placed between two numbers, it shows that addition is to be performed; thus, 3+4, is 7. The sign plus.

34. The sign of *equality* is =, and it signifies that the quantities between which it stands are *equal* to each other; thus, 3+4=7. The sign of equality.

35. These signs are not employed when the figures to be added are placed *vertically*; thus, When these signs are not used.

$$\begin{array}{r} 5 \\ 4 \\ 7 \\ \hline 16 \end{array}$$

36. The latter is the common form, and when the numbers are more than simple collections of units or ones, as in the example, it is, if not quite necessary, the most easy and convenient arrangement. When the vertical form is best.

37. Write the numbers to be added together in order, placing units under units, tens under tens, hundreds under hundreds, and so on. Draw a line beneath, as in the example, and if the amount or sum be less than ten, set it under that column; if it be ten or more, put down the units or ones as before, and add the tens to the next column. The sum of the last column is to be written in full. To add numbers.

38. Example 1.—Add together the numbers 50, 46, 68.

When the numbers have been placed as directed, the right hand or unit column is first added; thus, 8 and 6 and 0, are 14 units, or 1 ten and 4 units. The 4 is then put under the first or unit column, and the 1 ten is added to the second column, or the column of tens; thus, 1 and 6 are 7, and 4 are 11, and 5 are 16 tens, or 1 hundred and 6 tens. The 6 tens is placed Explanation of the way to add.

OPERATION.

$$\begin{array}{r} 50 \\ 46 \\ 68 \\ \hline 164 \end{array}$$

under the second column, or the column of tens, and the 1 hundred, which, if there had been another column, would have been added to that, is set at the left hand of the 6.

Sums for practice in addition.

	(2.)	(3.)	(4.)	(5.)
	58	5	5936	3650
	416	421	1862	2805
	233	3856	21	3217
Ans.	707	4282	7819	
	11	1 1	1 1	

Ans. (5.) left blank.

	(6.)	(7.)	(8.)	(9.)
	575	3360	56780	1309652
	8026	4527	2300	275943
	32	9802	45695	322568
	232	3765	37821	534576
	7495	4930	26308	323355
		2769	39376	486789
Ans.				

	(10.)	(11.)	(12.)
	3567820	5	39605067
	495323	74	4214503
	36532	532	7809
	4789	4789	890
	532	36532	20
	74	495323	5060
	5	3567820	25
Ans.			

PROOF.

Proof of sum.

39. To find if the columns have been added correctly, the simplest method is to observe the same process, only commencing at the top, and adding downward. If the figures correspond, it is the *proof* sought of correct addition; if they do not, the variation shows evidence of error.

A shorter way of adding.

40. In adding a column, omit the names of the figures. Thus, in the 12th sum, instead of saying 5 and 9 are 14 and 3 are 17 and 7 are 24, say briefly, 5, 14, 17, 24, and then setting down the 4 units* or ones, and carrying the 2, say 4, 10, 12, 21, 27, and so on. This is called *reading* the columns, and is done by good accountants. Frequent practice will render it easy.

Reading the columns.

* From the Latin *unus*, which means one.

EXAMPLE (13.) Add 7959, 6543, and 3487 together. Examples for practice in addition.

(14.) Add 6954, 78421, 5678, 34, 5659, 432178, 598765, 321234, 56789015, and 325 together.

(15.) Add 32, 361, 4500, 51189, 67891, 3666, 48572, 72584, 30605, 4336, 211667, 5876, 59865, and 999 together.

(16.) Add 7119, 59435, 625, 32, 59601, 436656, 5987451, and 23205 together.

(17.) Add 59590, 607805, 43021, 678015, 3676789, 39543201, 4856729, and 66 together.

(18.) Add 2005960, 5067520, 495610, 450, 3798510, 69450, 798504, 3296503, 576581, 723456, and 777 together.

(19.) Add 325678, 4353791, 56789012, 5391820, 506070801, 59694534, 3787956, 3950034, and 25 together.

(20.) Add 253651, 8135430, 376932, 6554322, 78985607, 359, 6432, 5765432, and 8 together.

(21.) Add five, fifty-five, one hundred and ten, one thousand and ten, two hundred and seventy-three, thirty-three millions, two thousand and ninety-nine, five thousand, and three hundred, and forty-eight together.

(22.) Add three trillions, two hundred and seven billions, five hundred and seventy-five millions, six hundred and thirty-six thousand, and seven hundred and eighty-four together.

(23.) Add three hundred and five billions, two hundred and forty-six millions, seven hundred and fifty-nine thousand and eighty together.

REMARK.—The sign that denotes a dollar, or dollars is $; thus, $1 represents one dollar ; $5, five dollars ; $100, one hundred dollars. The dollar sign

PRACTICAL EXAMPLES.

EXAMPLE (1.) There are due to my factors in Charleston, for money advanced on cotton, $450, to my grocers, $275, to the machinist for a steam-engine, $650, to the dry-goods merchants, $159, to the hardware dealers, $87, and to the Bank of the State of South Carolina, for a note discounted, $2895 : what is the amount of this indebtedness? Practical examples.

(2.) The State of Virginia had, in the year 1790, a population of 748,308 ; of Maryland, 319,728 ; of North Carolina, 393,751 ; of Tennessee, 35,791 ; of Kentucky, 73,077 ; of Georgia, 82,548 ; and of South Carolina, 249,073 : what was the aggregate? Population of Southern States in 1790.

(3.) The same States had, in 1830, the first named a population of 1,211,405 ; the second, 447,040 ; the third, 737,987 ; the fourth, 681,904 ; the fifth, 687,917 ; the sixth, 516,823 ; and the last, 581,185 : what was the whole number? Population of Southern States in 1830.

Free population of Confederate States in 1861.

(4.) The States forming the Southern Confederacy, in 1861, had, according to the last census—South Carolina, a free population of 301,271; Georgia, 595,079; Florida, 78,686; Alabama, 529,164; Mississippi, 354,699; Louisiana, 376,913; Arkansas, 324,233; Texas, 420,651; Virginia, 1,105,196: what was the total?

Slave population of the Confederate States in 1861.

(5.) The same Confederate States had, at that date, a slave population—South Carolina, of 402,541; Georgia, 460,232; Florida, 61,753; Alabama, 435,132; Mississippi, 436,696; Louisiana, 332,520; Arkansas, 111,104; Texas, 180,388; Virginia, 490,887: what was the entire number?

The United States and Confederate States sea-coast.

(6.) The United States has 9,334 miles of coast; the Confederate States, 23,803: what is the whole?

The sea-coast of the Northern States.

(7.) The State of Maine has of sea-coast and shores, of bays, sounds, &c., and of rivers to tide-head, a total of miles, 2,452

The State of		New Hampshire,	74
"	"	Massachusetts,	1,906
"	"	Rhode Island,	440
"	"	Connecticut,	1,327
"	"	New York,	2,057
"	"	New Jersey,	571
"	"	Pennsylvania,	106

What is the whole number?

Sea-coast of States South of Pennsylvania.

(8.) The State of Delaware has of sea-coast and shores, of bays, sounds, &c., and of rivers to head of tide, a total of miles, 671

The State of		Maryland,	4,453
"	"	Virginia,	2,372
"	"	North Carolina,	2,780
"	"	South Carolina,	1,256
"	"	Georgia,	924
"	"	Florida,	4,885
"	"	Alabama,	630
"	"	Mississippi,	385
"	"	Louisiana,	3,147
"	"	Texas,	3,069

What is the total?

(9.) The exports of the United States during the fiscal year of 1859, were, of domestic produce, chiefly from the slave States—

Exports of the United States in 1859.

Cotton,	$161,434,923
Rice and other vegetable productions,	24,046,752
Tobacco,	21,074,038
Manufactures,	33,853,660
Animal products,	15,549,817
Products of the forest,	14,489,406
" " sea,	4,462,974

What was the amount?

Imports of United States, 1859.

(10.) The imports of the United States, the same year, were,

Dutiable goods,	$259,047,014
Free goods,	72,286,327
Specie and bullion,	7,434,789
Domestic produce,	278,392,080
Foreign produce and merchandise.	14,509,971
Domestic specie and bullion,	57,502,305
Foreign " "	6,885,106

What was the amount?

Items of imports, 1859.

(11.) The imports into the United States during 1859 were—

Wool and woolens,	$37,966,910
Cottons,	29,830,364
Silks,	29,487,513
Flax and linens,	10,487,891
Tea,	7,388,741
Coffee,	25,085,696
Raw hides,	13,011,326
Tin plates,	5,331,147
Molasses,	5,062,859
Sugars, brown,	30,471,302
Tobacco and segars,	6,267,855
Iron,	9,088,239
Steel,	3,091,135
Brandy, wine, &c.,	8,919,885

What was the amount?

Population of the globe.

(12.) According to the late tables, the population of the globe in 1859, was—

Europe,	272,000,000
Asia,	755,000,000
America,	200,000,000
Africa,	59,000,000
Australia, &c.,	2,000,000

What was the entire number?

Population of the races

(13.) According to races, the population of the globe was, in 1860—.

Caucasian,	369,000,000
Mongolian,	522,000,000
Ethiopian,	196,000,000
American,	1,000,000
Malay,	200,000,000

What was the total?

Population religiously divided.

(14.) In religion, the population of the globe is thus divided,

Christians—Protestants,	89,000,000
Romish Church,	170,000,000
Jews,	5,000,000
Mohammedans,	160,000,000
Heathen,	788,000,000

What is the sum?

Losses of the British and Americans in the Revolutionary War.

(15.) In the war of the Revolution, the losses sustained by the English and Americans, including in some cases the wounded, and the surrenders at Saratoga and Yorktown, have been computed as follows:

	British. *Loss.*	American. *Loss.*
Lexington, April 19, 1775,	273	93
Bunker Hill, June 17, 1775,	1054	454
Fort Moultrie, June 28, 1776,	205	11
Flatbush, August 12, 1776,	400	200
White Plains, August 26, 1776,	409	400
Trenton, December 25, 1776,	1000	9
Princeton, January 5, 1777,	400	100
Hubbardstown, Aug. 16 and 17, 1777,	800	800
Bennington, August, 16, 1777,	800	100
Brandywine, September 11, 1777,	500	1100
Stillwater, September 17, 1777,	600	350
Germantown, October 4, 1777,	600	1200
Saratoga, Oct. 17, 1777, surrendered,	6752	——
Red Hook, October 22, 1777,	500	32
Monmouth, June 25, 1778,	400	130
Rhode Island, August 27, 1778,	260	211
Briar Creek, March 30, 1779,	13	400
Stoney Point, July 15, 1779,	600	100
Camden, August 16, 1779,	375	610
King's Mountain, October 1, 1780,	950	96

Cowpens, January 17, 1781,	800	72
Guilford Court House, March 15, 1781,	532	400
Hobkirk Hill, April 25, 1781.	400	400
Eutaw Springs, September, 1781,	1000	550
Yorktown, October, 1781, surrendered,	7072	

What was the entire loss on either side?

(16.) Gen. Washington was born A. D. 1732, and lived 67 years: in what year did he die?

(17.) How many days are there in the twelve calendar months, January having 31, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, December 31?

(18.) William the Conqueror began to reign in England in the year 1066, and reigned 21 years; William II., 13 years; Henry I., 15; Stephen, 39; Henry II., 35; Richard I., 10; John, 17; Henry III., 56; Edward I., 35; Edward II., 20; Edward III., 50; Richard II., 22. In what year was Richard dethroned?

(19.) From the creation of the world to the flood was 1656 years; from that time to the building of Solomon's Temple, 1336 years; thence to the birth of our Saviour, 1008 years: in what year of the world was our Lord born?

(20.) If you invested in the Bank of Georgia, $5000; in the Mississippi State Funds, $2550; in the Savannah City Stock, $1550; in the Southwestern Railroad Company, $2000; and in the Port Royal Railroad, $5500: what would be the amount thus placed?

(21.) A father in his will leaves to his eldest daughter, $5000; to his youngest, $3550; to his four sons, $300 each; and to them also, severally, the same sum left to his eldest daughter: what was the entire amount?

(22.) A planter sent to his factor, at one time, 4 bales of cotton, which weighed 1437 pounds; at another 6, which weighed 2100; at a later date 10, which weighed 3575; and afterwards 25, which weighed 8000: what was the whole number of pounds, and how many bales?

(23.) A factor received on consignment 300 bushels of rice; a few days later 2169; then 5560; afterwards 4175; and last, 2358: how many bushels in all?

(24.) There are 60 seconds in a minute, 3600 in an hour, 86,400 in a day, 604,800 in a week, 2,419,200 in a month, and 31,557,600 in a year: how many are all these combined together?

(25.) A merchant in Mobile received from New Orleans, ten casks of hardware, weighing each 1960+875+1000+

1268+999+2561+3248+1545+1862+1300: what was the weight altogether?

(26.) The sums on the debit side of a merchant's books for three different periods were as follows:

$5789.51	$2134.50	$789.61	
1643.42	4357.00	5.48	
347.31	429.66	14.52	
56.25	3906.25	23.07	832.68
3.33	6789.75	567.49	
78.16	1150.00	874.32	
1832.43	742.83	56.15	
741.50	5432.10	2.12	
250.30	212.13	75.15	
83.33	79.20	56.02	
16.79	8901.31	47.14	1678.39
7128.23	4728.53	2.25	
773.19	940.42	9.15	
940.43	2544.96	87.42	
59.75	366.03	569.87	
337.16	267.30	24.56	693.25
42.58	22.81	842.15	
18.76	256.00	47.96	
1530.21	55.02	81.15	
55.02	268.34	46.53	
586.75	367.35	487.20	
142.04	2269.54		1509.99

Ans.

The period mark distinguishing the dollars from cents.

Remark.—In the last three sums, the *period marks* between the second and third columns, distinguish the dollars and cents; thus, the bottom figures in the last sum, 1509.99, are read fifteen hundred and nine dollars, and ninety-nine cents.

Columns to be separated when very long

Note.—When the columns of figures are very long, let them be separated by lines, as in the last example; and the separated amounts placed by themselves, added for the required answer.

The census of the United States, in 1850, and that of 1860, were as follows: what were the totals?

CENSUS OF 1850.

(27.) STATES.	FREE.	SLAVE.	TOTAL.
Alabama,	428,779	342,844	771,623
Arkansas,	162,797	47,100	209,897
California,	92,597		92,597
Connecticut,	370,792		370,792
Delaware,	89,242	2,290	91,532
Florida,	48,135	39,310	87,445
Georgia,	524,503	381,682	906,185
Illinois,	851,470		851,470
Indiana,	988,416		988,416
Iowa,	192,214		192,214
Kansas,			
Kentucky,	771,424	210,981	982,405
Louisiana,	272,953	244,809	517,762
Maine,	583,169		583,169
Maryland,	492,666	90,368	583,034
Massachusetts,	994,514		994,514
Mississippi,	296,648	309,878	606,526
Missouri,	594,622	87,422	682,044
Michigan,	397,654		397,654
Minnesota,	6,077		6,077
New Hampshire,	317,976		317,976
New Jersey,	489,319	236	489,555
New York,	3,097,394		3,097,394
North Carolina,	580,491	288,548	869,039
Ohio,	1,980,329		1,980,329
Oregon,	13,294		13,294
Pennsylvania,	2,311,786		2,311,786
Rhode Island,	147,545		147,545
South Carolina,	283,523	384,984	668,507
Tennessee,	763,258	239,459	1,002,717
Texas,	154,431	58,161	212,592
Vermont,	314,120		314,120
Virginia,	949,133	472,528	1,421,661
Wisconsin,	305,391		305,391
TERRITORIES.			
Colorado,			
Dakotah,			
Nebraska,			
Nevada,			
New Mexico,	61,547		61,547
Utah,	11,354	26	11,380
Washington,			
Dis. of Columbia,	48,020	3,687	51,687
Ans.			

Census of the United States in 1850

Census of the U. S. Territories in 1850

Census of the United States in 1860.

CENSUS OF 1860.

(28.) STATES.	FREE.	SLAVE.	TOTAL.
Alabama,	529,164	435,132	964,296
Arkansas,	324,323	111,104	435,427
California,	380,015		380,015
Connecticut,	460,151		460,151
Delaware,	110.420	1,798	112,218
Florida,	78,686	61,753	140,439
Georgia,	595,097	462,230	1,057,327
Illinois,	1,711,753		1,711,753
Indiana,	1,350,479		1,350,479
Iowa,	674.948		674.948
Kansas,	107,110		107,110
Kentucky,	930,223	225,490	1,155,713
Louisiana,	376,913	332,520	709,433
Maine,	628 276		628,276
Maryland,	599,846	87,188	687.034
Massachusetts,	1,231,065		1,231.065
Mississippi,	354,699	436,696	791,395
Missouri,	1,058,352	114,965	1,173,317
Michigan,	749,112		749,112
Minnesota,	162,022		162,022
New Hampshire,	326,072		326,072
New Jersey,	672,031		672,031
New York,	3,887,542		3,887.542
North Carolina,	661,556	331,081	992.667
Ohio,	2,339,599		2,339,599
Oregon,	52.464		52,464
Pennsylvania,	2,906,370		2,906,370
Rhode Island,	174,621		174,621
South Carolina,	301,271	402,541	703,812
Tennessee,	834,063	275,784	1,109,847
Texas,	420,651	180,388	601,039
Vermont,	315,116		315,116
Virginia,	1,105,196	490,887	1,596,083
Wisconsin,	775.873		775.873
TERRITORIES.			
Colorado,	34,197		34,197
Dakotah,	4,839		4.839
Nebraska,	28.832	10	28.842
Nevada,	6,857		6.857
New Mexico,	93,517	24	93,541
Utah,	40,266	29	40,295
Washington,	11,578		11,578
Dis. of Columbia,	71,895	3.181	75,076

Census of the U. S. Territories in 1860.

Ans.

The primary mode of forming numbers by joining one unit to another, and the sum of those to a third, and so on, exhibits the principle of addition. When the numbers to be added consist of units of several degrees, such as tens, hundreds, etc., it is more convenient to add together the units of each order separately; and since ten units of any order make one unit of the next higher class, the number of tens in the sum of each order of units is carried to the next higher class and added to it. The principle of addition.

If the sum of the figures in each column be not in the excess of nine, the operation could be equally well commenced by the addition of the units of the highest order as by that of the simple units. But as it is oftener than otherwise the case, that several of these sums are in the excess of nine, to commence on the left obliges the operator to return back to correct a figure already written, and increase it by as many units as shall be obtained from the tens of the following column. Hence it is better in all cases to commence adding at the right. The reason for commencing addition at the right.

SUBTRACTION.

41. *Subtraction* is that operation which shows the *difference* between two numbers. Subtraction defined.

42. The *difference* when found, and *added to the smaller number*, gives the greater: this also *proves* the operation. The work showing the greater number; also, proof.

43. The difference is called the *remainder;* the greater number, the *minuend;* the smaller, the *subtrahend.* Names of terms.

44. The sign of subtraction (—) is called *minus*, and signifies *less.* When placed between two numbers, it shows that the right hand figure is to be taken from the one at the left; thus, 9—5=4, that is, nine minus five equals four: in this example 9 is the *minuend*, 5 the *subtrahend*, and 4 the *difference*, or as it is also called, the *remainder.* The sign minus. Example of its use.

45. To subtract a number, place the less beneath the greater, so that units come under units, tens under tens, hundreds under hundreds, and so on. To subtract numbers.

I. Commence at the right hand and subtract each figure of the lower line from the one directly above, when the one above is greater; then set the remainder under the horizontal line drawn beneath the subtrahend.

II. If the figure to be subtracted is the greatest, then

add 10 to the figure in the minuend, and proceed as above directed; always adding 1 to the next lower figure, when 10 has been added to the figure in the previous minuend.

Whence the 10 is taken.

REMARK.—The 10 which is added, when the unit figure is to be increased, is 1 ten from the next upper figure of tens; when the tens figure is to be increased, the ten is taken from the hundreds figure, and so on.

What is borrowing?

This is called borrowing, and in all cases when this is done, there must be carried or paid to the next subtrahend 1, which is the borrowed number in a different form, but of equal value.

EXAMPLE [1.]

Explanation of the work.

OPERATION.	
Minuend,	565
Subtrahend,	323
Remainder,	242

EXPLANATION.

In this example, we begin at the units column, and say 3 from 5 leaves 2. Setting that down, we then, at the second or tens column, say 2 from 6 leaves 4; and having put that down, we proceed with the last or hundreds column, and say 3 from 5 leaves 2, which, placed by the side of the 4 completes the sum.

	[2]	[3]	[4]	[5]
Minuend,	954	659	269	545
Subtrahend,	643	538	137	433
Remainder,				

Proof.

46. The answer to example 1 is found to be right, by adding the subtrahend and remainder together. These when added give the minuend:

Subtrahend,	323
Remainder,	242
Minuend,	565

EXAMPLE [6.]

Explanation

OPERATION.	
Minuend,	834
Subtrahend,	427
Remainder,	407
Proof,	834

EXPLANATION.

Here, we cannot take 7 units from 4 units, but by adding 10, which makes the minuend 14, we can say 7 from 14 leaves 7. This being put down, we carry 1 to the next figure, and say 3 from 3 leaves naught. When this has been set down, we proceed with the last figure, and say 4 from 8 leaves 4, and place it beside the last. The sum is then done, and is proved by the addition of the subtrahend and remainder.

	[7]	[8]	[9]	[10]	[11]	[12]
Min.	5965	3256	6915	9654	7643	5395
Sub.	4876	1337	5938	8573	6374	4236
Rem.						
Pr.						

EXAMPLE [13.]

OPERATION.

Minuend,	7650
Subtrahend,	5761
Remainder,	1889

EXPLANATION.

In this example, with the figure in the thousands-place, all of the subtrahend is greater than the minuend. Of course 10 has to be borrowed and added to each figure of the minuend, except the last, and 1 is to be carried or paid to every figure in the subtrahend, except that with which the sum is commenced. Thus, we say 1 from 10 leaves 9; then carrying 1 to the 6, we say 7 from 15 leaves 8; carrying 1 to the 7, we say 8 from 16 leaves 8; and last, 1 to 5 is 6, and 6 from 7 leaves 1. Explanation

	[14]	[15]	[16]	[17]	[18]
Min.	7865	8954	7532	88542	1045679
Sub.	976	6165	4507	59653	565789
Rem.					
Pr.					

	[19]	[20]	[21]	[22]	[23]
Min.	3653456	42310105670	1000	60000	45670000
Sub.	2895667	37824306580	1	99	432506
Rem.					
Pr.					

24. From 56754302, take 4356706.
25. From 65374231, take 5900327.
26. From twenty thousand and thirty-five, take one thousand five hundred.
27 From one billion, two hundred and forty-two thousand and twenty-three, take one million and ninety-five.
28. 376326945—22654178= ?
29. 456897003—3955487= ?
30. 90005434—785070= ?
31. 7000000—3999993= ?
32. 58675432—40101011= ?
33. In a certain example, the minuend is 368 and the subtrahend 209: what is the remainder?
34. Henry Hudson sailed up the North river, now called after that navigator, the Hudson, in 1609: how many years since?

35. Suppose you should borrow of a friend $2000, and pay back at different times $1695: how much is the remaining indebtedness?

36. If your income is $2500 a year, how much is your annual deficiency if your expenditures amount to $2745?

37. The battle of Fort Moultrie was fought in 1776: what number of years have elapsed since that memorable contest and the investment and capture of Fort Sumter, in 1861?

38. The Revolutionary war commenced in 1775: how long is it since that date and the war of 1812?

39. The distance from the earth to the sun is called 95,000,000 miles; the distance to the moon 240,000: how many more miles is it to the sun than to the moon?

40. South Carolina passed the ordinance of secession, December, 1860: how many years is that era from the discovery of America in 1492?

REMARK.—Sometimes it is convenient to subtract without placing the subtrahend beneath the minuend. It is, however, virtually so placed.

Subtrahend,	957
Minuend,	2756
Remainder,	1799

41. Subtract 957 from 2756.

42. Subtract 635 from 1650.

43. Subtract 342 from 560.

44. Subtract 541 from 9650.

The rationale of subtraction.

Subtraction is performed by taking the units of each degree in the subtrahend from those of the corresponding degree in the minuend, and severally denoting the remainders. When the units of any degree in the subtrahend exceed those of the same degree in the minuend, one unit of the next higher degree is to be mentally joined to the deficient place in the minuend, and the units of the higher degree to be considered one less than denoted. Another method may be adopted in this case: increase both the minuend and subtrahend by the mental addition of ten to the deficient place in the former, and one to the next higher degree of units in the latter. This method is justified by the self-evident truth that if two unequal quantities be equally increased, their difference is the same.

An example and explanation.

To find the difference which exists between two numbers, when some figures in the lower line are greater than some in the upper, the following process is to be observed:

Having arranged the numbers, as in the example given, we say 4 from 6 leaves 2, which is written under the units. Passing to the column of tens, as the lower figure 8 is greater than the upper one 5, it cannot be subtracted. To overcome this difficulty, we borrow mentally from the hundreds figure 1 hundred, which is equal to 10 tens, and add to it the 5 tens, which we have already, making it 15 tens; we then say 8 from 15 leaves 7, which is written in the column of tens. Proceeding to the column of hundreds, we observe that the upper figure ought to be diminished by 1, as this unit was borrowed in the preceding subtraction: we say, then, 7 from 3 (or by adding, which would be equivalent, the borrowed 1 to 8, we say 8 from 4), which is impossible; but we borrow, as before 1 thousand, which equals 10 hundreds, giving 13 hundreds, and take 7 from 13 which leaves 6 (or by adding the borrowed unit to the 7, and taking 8 from 14), to be placed in the column of hundreds. Passing to the thousands, 8 cannot be taken from 2 (or, equivalently, 9 from 3), but 8 from 12 leaves 4 (or 9 from 13), which is written in the column of thousands. Lastly, as the figure 8, of tens of thousands, on account of the 1 just borrowed ought to be replaced by 7, we say 2 from 7 leaves 5 (or, equivalently, 3 from 8). Thus the remainder, or the excess of the greater number over the less, is 54672.

OPERATION.

$$\begin{array}{r} 83456 \\ 28784 \\ \hline 54672 \end{array}$$

The rationale.

To understand how by this method we learn such result, it is sufficient to state that the two numbers could be thus arranged:

	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
1st number,	7	12	13	15	6
2d number,	2	8	7	8	4
	5	4	6	7	2

It thus appears that the upper number exceeds the lower one by 2 units, 7 tens, 6 hundreds, 4 thousands, and 5 tens of thousands—or exceeds it 54672 units.

A second example.

Again, to subtract the number 158429 from 300405.

OPERATION.

$$\begin{array}{r} 99\ 9 \\ 300405 \\ 158429 \\ \hline 141976 \end{array}$$

Rationale.

As 9 the units figure of the lower number is larger than 5, the corresponding figure of the greater, we borrow 1 ten from the first figure to the left, but this figure being 0, it is necessary to have recourse to

the figure 4 of hundreds, from which 1 is borrowed, equal to 10 tens; but as only a single 1 is needed, we leave 9 of them above the 0; 1 ten is then added to 5, which gives 15; we then say 9 from 15 leaves 6, which is written under the units. Proceeding to the tens, we say 2 from 9 leaves 7. For the hundred, as the upper figure 4 has been diminished by the borrowed 1, and as 4 cannot be taken from 3, recourse is had to the next figure on the left; but that and the figure to the left being the cipher 0, 1 is borrowed from the next significant figure, 3. This 1 equals 10 of the order following and 100 units of the order thousands; since, however, we have need of only 1 unit of this order, we leave 99 of them which are placed above the two ciphers; adding 1 thousand to the 3 hundreds, it becomes 13 hundreds, and we say 4 from 13 leaves 9, which is placed under the column of hundreds.

In the two following subtractions, each one of the ciphers being replaced by a 9, we say 8 from 9 leaves 1, and 5 from 9 leave 4. Passing to the next, we say 1 from 2—the figure 3 being diminished one—leaves 1, or what is equivalent, by adding what is borrowed to the lower figure, 2 from 3 leaves 1.

The operation could be thus arranged:

	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
1st num.	2	9	9	13	9	15
2d num.	1	5	8	4	2	9
	1	4	1	9	7	6

Thus the greater number exceeds the less by 6 units, 7 tens, 9 hundreds, 1 thousand, 4 tens of thousands, 1 hundred thousand, or by 141976 units.

A more convenient plan.

Note.—The customary and the better plan is, instead of diminishing by one unit the figure from which we borrow, to leave this figure unchanged, but augment the corresponding figure below by one unit.

EXERCISES IN ADDITION AND SUBTRACTION.

Exercises on the rules of addition and subtraction.

47. EXAMPLE 1.—If a man's income is 3500 dollars a year, and he spends 500 dollars for house rent, 950 dollars for sundry domestic expenses, 400 dollars for travelling, and 350 dollars for charitable purposes: what will remain to him when the year is up?

2. How many are 5678+7891+6035+22+9658—595?

3. What is the sum of $3650+$7845+$945+$736+$814+$7690—$2450?

4. How much are \$334+\$565+\$1890, \$37—\$259+\$67—\$20—\$6?

5. A merchant gains by trading at one time \$567, but loses by a bad debt \$160; at another time he gains \$675, but loses \$369; at a third time he gains \$2500, but loses \$1535: what remains from his gains and losses?

6. A grocer in Louisville purchased of a Charleston house 75 hogsheads of molasses, for 25650 dollars, paid expense of transportation, 365 dollars, and then sold the same for 950 dollars less than it cost him: how much did he receive?

7. If a trader were to buy articles for \$650, and sell \$250 worth of them to one man, and \$125 to another, what would be the amount remaining for sale?

8. A farmer bought 200 sheep, and gave 300 dollars for them; a yoke of oxen, which cost 65 dollars; a horse, 125 dollars; a cow, 40 dollars; and he paid toward the purchase 100 bushels corn, valued at 75 dollars; 200 bushels oats, at 80 dollars; 800 weight of blades, at 50 dollars, and gives his note for the balance: what is the amount of his note?

9. If you purchased 100 oranges for 250 cents, 200 limes for 538 cents, 5 bunches of bananas for 950 cents, and 25 cocoa-nuts for 150 cents, what would they come to in cents? what in dollars and cents? what, if for bad fruit, 338 cents were taken off, would the amount be?

10. In France, during the Reign of Terror, there were guillotined, by sentence of the Revolutionary Tribunal, nobles 1278, women of the same class 750, wives of artisans 1467, religieuses 350, priests 1135, persons not noble 13,623; what was the entire number? What is the difference between this and the aggregate of other victims put to death in other forms?—women killed in La Vendee 15,000, women dying from grief and fear 3,748, children killed 22,000, men 900,000, victims at Lyons 31,000, at Nantes 32,000.

MULTIPLICATION.

48. *Multiplication* is a process by which is found, quickly, the amount of a given number. Multiplication defined.

49. The given number is called the *multiplicand*; the one which is used to discover the required amount is called the *multiplier;* and the answer found is called the *product*. The multiplicand and multiplier are called *factors*—that is the *makers* of the product. Names employed: Multiplicand. Multiplier, Product, Factors.

The sign X. **50.** The sign $\times$ placed between two numbers, denotes their *multiplication* together; the *result* of the multiplica-

What is a compound number. tion is known as a *compound number:* thus, $6\times3=18$ shows, that by the use of the factors, 6 and 3, the *result*, or the

Use described. *compound number* 18 has been found.

The same product though the figures are reversed. **51.** The product or result is invariably the *same*, whether the factors stated as above, or in the reverse order, 3×6.

To facilitate multiplication. **52.** To facilitate multiplication, it is necessary to keep in memory the sum of each of the nine first numbers, or digits, as they are called,* repeated from one time to nine times; that is, the products of each of the nine digits by themselves and by each other. The common tables usually extend through 12; and the following copies them, as it is convenient to know more than the products of 9×9.

MULTIPLICATION TABLE.

The multiplication table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

* So named from an old custom of counting upon the fingers, and derived from the Latin word *digitus*, a finger.

The multiplication table described.

53. This table was made by writing, as in the upper row, the numbers 1, 2, 3, 4, etc. The second by adding these numbers to themselves, and writing them directly under the first; thus 1 and 1 are 2; 2 and 2 are 4; 3 and 3 are 6, etc. The third row by adding the second to the first; thus 2 and 1 are 3; 4 and 2 are 6; 6 and 3 are 9, etc.; this contains the first row, it will be observed, three times. The fourth row is formed by adding the third row to the first, and so on with the rest.

The mode of its use.

54. When the formation of the table is comprehended, the mode of use will be apparent. If, for instance, the product of 8 by 5, that is 5 times 8 were required, we look for 8 in the upper row, then directly under it, in the fifth row, is found 40, which is 8 repeated 5 times. In like manner, we find other numbers.

Multiplication is repetition, or a short way of adding.

55. To multiply one number by another, is to *repeat*—as is done in addition—the first number as many times as there are units, or ones, in the second number.

Multiplication performed the two methods.

EXAMPLE 1.—In one pasture are 8 sheep: how many are there in 5 pastures, each with the like number?

BY ADDITION.	BY MULTIPLICATION.
8	
8	
8	
8	8
8	5
Ans. 40	*Ans.* 40

2. A field is marked out into 12 tasks: how many tasks would there be in 12 fields similarly divided?

Explanation of the work in the given example.

OPERATION.		EXPLANATION.
Multiplicand,	12	Multiply first by 2, as though 2 were the only multiplier; then by 1, and set the first figure of the result in the place of tens, or the second column,* and then the 1 by the 1, and set it in third column, or that of hundreds.
Multiplier,	12	
	24	
	12	
Product,	144	

Then draw a line beneath, add the two results, and the product will be as shown in the example.

The same process always.

56. The same process is to be observed whether the example have few or many figures.

* ~~We use the word column to avoid repetition of the word place, and for want of a better term.~~

3. In a day are 24 hours: how many hours are there in 28 days?

Explanation of work.

OPERATION.		EXPLANATION.
Multiplicand,	24	In this example, we first say 8
Multiplier,	28	times 4 is 32, and then place the
	——	2—a line being drawn—under the
	192	8, or in the units column. We
	48	then say 8 times 2 is 16, and add
	——	to that the 3 of the first multipli-
Product,	672	cation, which making 19, is set at

the left of the 2, the whole being 192. We then proceed to the second figure 2 in the multiplier, and say 2 times 4 (or preferably twice 4) is 8, which is put down in the second or tens column; and as nothing remains over to be carried, we proceed to the next and say, 2 times (or twice) 2 is 4. This is placed at the left of the 8 in the hundreds column, a line beneath drawn, and the results added for the product, as above.

A partial product.

57. The product obtained by the multiplication of a single figure of the multiplier is called a partial product. In the last example, 192 and 48 are such.

58. From these examples, we draw the following plain directions for multiplying a number:

Direction for multiplying numbers.

I. Place the multiplier under the multiplicand, and draw a line beneath.

II. Commence at the right hand, and multiply the multiplicand by each figure in the multiplier, and place the first figure of each partial product directly under its multiplier, and the others in order at the left.

III. When the multiplication is completed, draw a line beneath, and add for an answer the results of the multiplication.

Proof.

59. To prove a sum, simply reverse the position of multiplicand and multiplier, and proceed by the directions.

4. Multiply 4320 by 345.
5. Multiply 58760 by 4632.
6. Multiply 897432 by 3691.
7 Multiply 94870 by 3323.
8. Multiply 660761 by 4232.
9. Multiply 3010102 by 6654.
10. Multiply 34208 by 6985.

When a cipher is in the multiplier.

60. When a cipher occurs between the figures of the multiplier, it must, as much as any other figure, be set in its place in the partial product.

11. Multiply 9567 by 8005.

Example of intermediate ciphers.

9567
8005
47835
7653600
Ans. 76583835

Note—This simply shows the value of ciphers in certain positions.

12. Multiply 90574 by 8903.
13. Multiply 79876 by 7065.
14. Multiply 37965 by 2002.
15. Multiply 54032 by 2561

REMARK.—It is to be observed, that 0 multiplying, or multiplied by a number, 5 for instance, is 0, and will, if it be the multiplier, form a partial product, unless the briefer plan, as shown in the 11th example, be adopted. To explain the point in the following example, both methods are given. The briefer is the better.

16. Multiply 3665 by 5002.

3665
5002
7330
18325000
Ans. 18332330

3665
5002
7330
0000
0000
18325
Ans. 18332330

61. When ciphers ~~are at~~ the right hand, the same can be extended beyond their usual place, and simply brought down, before we commence with the multiplication of the next figure.

A cipher or ciphers at the right hand.

17 Multiply 4567 by 300.

4567
300
Ans 1370100

18. Multiply 90700006 by 80507700

19 Multiply 51060504 by 6530050

20. Multiply 69007 by 2035000

21. Multiply 807065 by 203040

22. Multiply 357555 by 23600

23. Multiply 264 by 389000

24. Multiply 3542 by 76500

25. Multiply 28534 by 65209000

26. 44444×55555= ?

27. 777777×88888= ?
28. 875643×345678= ?
29. 975310×13579= ?
30. 76543167×5943478= ?

62. The directions already given can be used in all examples of multiplication; but since it may be desirable at times to shorten the process, we have the following direction:

A way to shorten the multiplication of compound numbers.

63. When the multiplier is a compound number, multiply the multiplicand by one of the factors of the multiplier, and that product by another factor, and so on, until all the factors have been employed. The last product will be the answer.

31. Multiply 54 by 21.

Example of factoring multiplication.

OPERATION.

21=7×3.

Multiplicand,	54
1st Factor of Mul.,	7
	378
2d Factor of Mul.,	3
Product,	1134

REMARK.—The factors of 21 are the numbers 7 and 3; and it is plain that 7 times 3 times a number, are 21 times that number.

32. Multiply 355 by 40, or its factors 8 and 5.
33. Multiply 6789 by 63.
34. 565×72=how many?
35. 36789×42=how many?
36. 96569×36=how many?
37 23467×54=how many?

How to dispose of ciphers when in multiplicand and multiplier.

64. When ciphers are at the right hand both of the multiplicand and multiplier, it is sufficient to multiply in the ways already stated, and then to place the full number of ciphers at the right of the product.

38. Multiply 39500 by 65000.

```
        395
         65
       ----
       1975
      2370
      ----------
Ans.  2567500000
```

39. Multiply 4500 by 6500.

```
       4500
       6500
       ----
        225
       270
       --------
Ans.   29250000
```

40. Multiply 345600 by 28700.
41. 26900×34300.
42. 6785600×353200.
43. 27000×690.
44. 53600×70500.
45. 840320×543000.
46. 2689000×135600.
47 901000×75300.

PRACTICAL EXAMPLES.

Practical examples in multiplication.

1. What is the cost of 10 pounds of coffee at 15 cents a pound?

2. What is the value of 350 bushels of corn at 75 cents a bushel?

3. There are 320 rods in a mile: how many rods are there in 265 miles?

4. Suppose the Charleston Mercury to have 32 columns, and 16 paragraphs on an average in each column: how many paragraphs would there be in the paper?

5. Suppose the same paper to have 9 words in each line of 512 paragraphs: what number of words would there be?

6. If a regiment contain 780 men: what, if they were of like number, would there be in 120 regiments?

7. What is the value of 250 acres of land, at $36 an acre?

8. In its annual revolution, the earth moves about 19 miles the second: how far does it move in a week, supposing that in a week are 604,800 seconds?

9. How many pounds of cotton in 432 bags, each bag containing 337 pounds?

10. Sixty-five men can build a wall in 45 days: how long would it take one man to build a wall 18 times as long?

11. Two men travel in different directions; one at the rate of 65 miles a day, the other 45: how far will they be apart in 15 days?

12. What must I pay for 12 barrels of flour, at $7 a barrel; 250 bushels of rice, at $3 a bushel; 5 barrels of molasses, at $18 a barrel; and 3 hogsheads of bacon, at $75 a hogshead?

Multiplication an abridged method to find a sum.

Multiplication is simply an abridged method of finding the sum of several equal quantities by the repetition of one of those quantities.

The rationale of multiplication.

When the product of factors, consisting of several figures, is required, it is necessary to multiply each figure in the multiplicand by each figure in the multiplier, and denote the several products in such order that they shall represent their respective values. When simple units are employed as the multiplier, the product of each figure in the multi-

plicand is of the same degree as the figure multiplied; that is, units multiplying units give units, units multiplying tens give tens, units multiplying hundreds give hundreds, etc. When tens are employed as the multiplier, the product of each figure in the multiplicand is one degree higher than the figure multiplied; that is, tens multiplying units give tens, tens multiplying tens give hundreds, tens multiplying hundreds give thousands, etc. When hundreds are employed as the multiplier, the product of each figure in the multiplicand is two degrees higher than the figure multiplied, and so on.

OPERATION.

To multiply	87468
By	5847
	612276
	3498720
	69974400
	437340000
	511425396

We commence by placing the multiplier under the multiplicand, so that the units of the same order fall in the same column. This being arranged, we observe that to multiply 87468 by 5847, is to take the multiplicand 7 times, 40 times, 800 times, and 5000 times; then to add together these partial products. We can first find the product of 87468 by 7, which gives 612276.

The second operation reduces itself to multiplying the multiplicand by the figure 4, considered as expressing simple units in writing a 0 to the right of the product, and in placing the result, as in the written operation, below the first partial product. In like manner, in order to perform the multiplication of 87468 by 800, it is sufficient to multiply that number by 8, which gives 699744, then annex two ciphers to the right of this product; we thus have a third partial product, which is placed below the two preceding products. So also to perform the multiplication of 87468 by 5000, it suffices to multiply by 5, to annex three ciphers to the product and write the result, 437340000, below the first three products. Adding the four partial products, we have the total product 511425396.

In the multiplication of the 4, or the tens figure in the above, we may conceive that we have written one under another, 40 numbers, each one being 87468, and that we add them to obtain the required product. It is, however, evident that these 40 numbers form ten divisions, each containing 4 times 87468. Adding and multiplying the result by 10, or what is equivalent, annexing a 0, we obtain 3498720 for the product of 87468 by 40.

A similar reasoning applies to the third number 8; for

800 numbers equal to 87468, and placed one under another, form, evidently, 100 divisions of 8 numbers, each equal to 87468, or 100 numbers, equal to the product of 87468 by 8; that is, 6997400: also, to the fourth number 5, that 5000.

It is customary to dispense with the ciphers to the right of the partial product; but we write each partial product below the preceding one, advancing it one place to the left; that is, we make the first figure multiplied occupy the same column which the figure by which we multiply, occupies. The dispensation of the use of ciphers.

To determine if an error has happened in the process of multiplication, the following method of trial, which depends on the peculiar property of the number 9, and which is called casting out the nines, may be practised: To determine error. The casting out of the nines.

Add together the figures of the product horizontally, rejecting the number 9 as often as the sum amounts to that number, and proceeding with the excess, and finally denote the last excess. Perform the same operation upon each of the factors; then multiply together the excesses of the factors, and cast out the nines from their product. If the excess of this smaller product be equal to the excess of the larger product first found, the work may be supposed to be right. Such test is, however, not infallible; for if a product happen to contain an error of just 9 units of any degree, the excess of its horizontal sum is not thereby altered. The method.

To understand why the excess above nines found in the horizontal sum of a product, must be equal to the excess found in the product of the excesses of the factors, it is to be noticed that by the law of notation, a figure is increased nine times its value by its removal one place to the left; and hence, however far a figure is removed from the place of units, when its nines are excluded, its remainder can only be itself. Hence, any number, and the horizontal sum of its figures, must have equal remainders when their nines are excluded. This being understood, let it be observed, that since factors composed of entire nines will give a product consisting of entire nines, it follows that any excess above nines in a product must arise from an excess above nines in the factors. Therefore, the product of the excesses of the factors must contain the same excess that is contained in the product of the whole factors. The reason why.

DIVISION

Division defined.

65. *Division* is a process used to find how many times one number is contained in another.

Names employed in division.

66. The first number used for this purpose is called the *divisor;* the second is called the *dividend;* and the third, or result, is called the *quotient.*

The overplus or remainder.

67. When any number is *over* to the performed division, it is called the *remainder.*

Signs of division.

68. Three signs are employed to express division, namely, ÷ ; —) ; and are thus used, as in the case where 15 is to be divided by 5 ; 15÷5 ; $\frac{15}{5}$; 5)15.

When the divisionary curve)(is used.

69. When the divisor exceeds 12, and the divisionary curve) is used, one is drawn corresponding to it (on the right of the dividend, and the quotient placed against it; thus, 15)135(9 quotient.

Division of two kinds.

70. *Division* is distinguished as *short* and *long.*

Short division.

71. *Short division* is the method used when the results simply are written, in consequence of the divisor being 12, or less than 12; for its performance observe the following directions:

Directions for Short division.

72. I. Write the divisor on the left of the dividend. Begin at the left, and having divided the figure, or the fewest figures in the dividend, that contain the divisor, set each quotient figure under its dividend.

II. When there is a remainder after any division, annex to it the next figure of the dividend and proceed as before.

III. Should any figure of the dividend be less than the divisor, put down 0 for its quotient, and annex the next figure of the dividend for a new dividend.

IV Should there be a remainder after the division of the last figure, set the divisor under it, and annex the result to the quotient.

Proof.

73. To discover if the work is correct, multiply the divisor by the quotient, and if there is a remainder add it to the product. This will give the dividend if the division has been correctly performed.

EXAMPLE 1.

Divisor, 3)6396, Dividend.

Quotient, 2132
3

Proof, 6396

2. Divide 7340 by 5.

OPERATION.	EXPLANATION.
Divisor, 5)7340, Dividend. Quotient, 1468 5 Proof, 7340	In this sum we say 5 into 7, 1 time (or, preferably, once) and 2 over; setting down the 1 under the 7, and annexing the 2 that was over to the 3, we say 5

into 23, 4 times and 3 over; setting down the 4 under the 3, and annexing the 3 that was over to the 4, we say 5 into 34, 6 times and 4 over; setting the 6 under the 4, and annexing the 4 to the 0, we say 5 into 40, 8 times, which placed under the 0, concludes the sum. Explanation of work of operation.

3. Divide 8940 by 5.
4. Divide 42340 by 4.
5. Divide 76212 by 6.
6. Divide 89754 by 7.
7. Divide 95436 by 9.
8. Divide 676740 by 3.
9. Divide 27840 by 5.

Note.—As in this example, the divisor 5 is not contained in the first figure 2, of the dividend, we say 5 into 27, and then proceed. When the divisor exceeds the first figure of the dividend.

10. Divide 56123456788 by 12.
11. Divide 456789103277 by 11.
12. Divide 768492010340 by 10.
13. Divide 567894321 by 9.
14. Divide 64765434 by 8.—*Ans.* 8095679 and 2 over.
15. Divide 3245675 by 5.
16. 678524÷4= ?
17. 78954÷5= ?
18. 323506÷8= ?
19. 5437008÷6= ?
20. 3390005÷9= ?

LONG DIVISION.

74. When the divisor exceeds 12, it is customary to employ the method called *Long Division*. In this, the entire work is put down; the mental calculation, as in short division, being dispensed with. When long division is used.

75. To perform Long Division—

1. Write the divisor and dividend as in short division, and draw a curve on the right of the dividend.

How to perform long division.

II. Divide the smallest number of figures in the left of the dividend, that will contain the divisor, and set the result, as the first figure of the quotient, at the right of the dividend.

III. Multiply the divisor by each new quotient figure, and set the product under that part of the dividend taken for division.

What is a partial dividend?

IV Subtract the product from the figures over it, and to the remainder annex the next figure of the dividend for a new partial dividend.

V Divide the partial dividend, and proceed as before, until the whole dividend is exhausted.

21. Divide 46755 by 15.

Explanation of the work.

OPERATION.

```
15)46755(3117 Ans.
   45
   —
    17
    15
    —
     25
     15
     —
     105
     105
     ———
```

EXPLANATION.

In this example, we say 15 into 45, 3 times, which 3 is put in the quotient; then we multiply the divisor by the 3, and set the result, 45, under the 46 in the left of the dividend; we next subtract the 45 from the 46, and having drawn a line beneath the 45, put the remainder, 1, under the 5, and draw down the next figure in the dividend, 7; we then say 15 into 17, 1, and putting this 1 in the quotient, multiply the divisor by the 1, and place the 15 under the 17; drawing a line beneath, we subtract the 5 from the 7, and to the 2 annex the next figure 5 in the dividend, and so on until the division is completed.

22. Divide 678954 by 25.
23. Divide 78956 by 35.
24. Divide 954321 by 45.
25. Divide 345687 by 50.

When the divisor contains three or more figures, sometimes it is difficult to discover how many times the divisor is contained in the figures separated as a partial dividend. Often the difficulty is obviated or decreased by trying the first figure of the divisor into the first figure or first two figures of the partial dividend. Such trial indicates nearly the true figure.

26. Divide 436940074 by 64237

OPERATION.	EXPLANATION.
64237)436940074(6802 *Ans.* 385422 ——— 515180 513896 ——— 128474 128474 ———	In this example, we find how many times 6, the first figure of the divisor, is contained in 43, the first two figures at the left of the dividend. It is found to be 7, and 7 is contained 6 times. This is the limit or extent. By trial, we find

it cannot be 7, for 7 times 6 are 42, which subtracted from 43, leaves 1 to be joined to the next figure 6, for a new partial dividend. But 4, the second figure of the divisor, is not contained 7 times in 16, therefore 6 and not 7, will be the first figure in the quotient.

27. Divide 56679670 by 3456.
28. Divide 89765432 by 94432.
29. Divide 7543215 by 876504.
30. Divide 37651245 by 568912.
31. Divide 29543278 by 896302. 32. 35678÷412= ?
33. 56945÷375= ? 34. 88003÷510= ?

76. When the divisor is 10, 100, 1000, etc., cut from the right hand side as many figures as there are ciphers in the divisor. The figures at the left will be the quotient, and those at the right the remainder.

35. Divide 846 by 10. *Ans.* 84–6 or 84 quotient and 6 remainder.
36. Divide 95607 by 10.
37. Divide 74568 by 10.
38. Divide 76543 by 100. *Ans.* 765–43.
39. Divide 234678 by 100.
40. Divide 35987654 by 1000. *Ans.* 35987–654.
41. Divide 59761120 by 1000.
42. Divide 9396781 by 1000.

77. When the divisor is a compound number (Art. 59), a short method to find the quotient is to separate the divisor into factors, and proceed as in the following:

43. Divide 5626800 by 40, that is, by its factors 5×8=40.

OPERATION.

1st Factor, 5)5626800, Dividend.

2d Factor, 8)1125360, 1st Quotient.

140670 2d Quotient, or Result.

44. Divide 1678 by 24.
45. Divide 1895 by 35.
46. Divide 3564 by 54.
47. Divide 6780 by 56.
48. Divide 32960 by 21.

Division by 3 factors.

78. When the divisor can be resolved into 3 factors the like process is to be performed, thus:

Work performed.

49. Divide 990576 by 108, or by the factors $9\times6\times2=108$.

OPERATION.

```
9)990576
 ------
6)110064
 ------
 2)18344
  -----
   9172  Quotient.
```

50. Divide 187236 by 252.
51. Divide 1255872 by 192.
52. Divide 12393327 by 189.

When there are remainders to factoring divisors.

79. Should there be remainders to the different divisions, multiply the last remainder by the last divisor but one, and add in the preceding remainder; then multiply this result by the next preceding divisor, and add in the remainder, and so on until the first remainder is added; the sum obtained in this way will be the true remainder.

If the remainder is a cipher.

Note.—Should any remainder be a cipher, then let a cipher be added.

53. Divide 15956 by 280, that is, by the factors $7\times5\times8=280$.

OPERATION.

```
7)15956
  -----
 5)2279–3 Remainder.
   ----
  8)455–4 Rem.
    ---
     56–7 Rem.
```

Work showing the true remainder.

True remainder as seen by the following division:

```
280)15956(56
    1400
    ----
     1956
     1680
     ----
      276
```

By application of method (Art. 79), $7\times5+4=39$; $39\times7+3=276$, the true remainder.

54. Divide 3765452 by 126.

55. 4547801÷140= ?
56. 5137659÷1156= ?
57 How many times is 310 contained in 75643 ?
58. What would you divide 587654 by, to have for its quotient 475 ?
59. If a dividend is 694505, the quotient 3006, and the remainder 119: what is the divisor?
60. If a dividend is 870654, and the divisor 320, what are the quotient and remainder ?

89. Sometimes it is more convenient to place the divisor at the right of the dividend, and the quotient beneath, thus: The divisor placed at the right hand.

61. Divide 578934 by 28.

```
578934|28
      |------
56     20676-6 Ans.
--
 189
 168
 ---
  213
  196
  ---
   174
   168
   ---
     6
```

62. Divide 89765643 by 29.
63. Divide 53740912 by 356.
64. Divide 9655468 by 4678.

Division, which is the converse of multiplication, not only investigates the number of times the dividend contains the divisor, but it also serves to divide the dividend into as many equal parts as the divisor contains units, the quotient being one of the given parts. The object of division.

Division must begin at the highest degree of units, for the number of times that the divisor is contained in the higher units of the dividend must be taken out first, in order that any remainder, or excess above an exact number of times may be carried down to the lower degree of units and divided therewith. When the divisor is not contained an exact number of times in the dividend, there will be a remainder at the end of the operation. This remainder is a part of the dividend, and is to be divided; but its quotient will be smaller than a unit, since a quantity in the dividend just equal to the divisor gives only a unit in the quotient. Why division commence at the highest degree of units.

Why in division the work is begun at the left.

In the first three operations of arithmetic, the calculations are performed by commencing at the right; in division we commence at the left, because the dividend being the sum of the partial products of the divisor, by the units, tens, hundreds, etc., of the quotient, all these partial products are mingled one with another, so that it is impossible to commence by separating out the product by the units, by the tens, etc. By the established method, we are able to determine at once in what part of the dividend the product by the highest units is found, and then we obtain the figure of the highest units, and then arrive at the figure of the units of the order immediately below the first, and so on.

Rationale when any number of figures is given.

When the dividend, divisor and quotient contain any number of figures, as in the accompanying sum, the following explanation is given:

9176298	2678
11422	3426
7109	
17538	
1470.	

Placing mentally three zeros, and afterwards four zeros to the right of the divisor, we obtain two products, 2678000, and 26780000, the one less, the other greater than the dividend. Thus, the total quotient is comprised between 1000 and 10,000, or must be composed of four figures, of which the first to the left expresses *thousands*. In order to find this first figure, we observe that its product by the divisor, inasmuch as it is thousands, is to be found wholly in the part 9176 thousands of the dividend. We are then led to divide 9176, which we consider as a first partial dividend, by 2678; and the greatest number of times that the second number is contained in the first represents the *thousands* figure of the whole quotient. Now, the true quotient of 9176 by 2678, obtained according to the method of trial indicated, is 3. We write, then, 3 below the divisor. Next, we subtract from the dividend the product of the divisor by 3, either by placing this product below the partial dividend, and subtracting or effecting simultaneously the subtraction and the multiplication as above. This first operation amounts, evidently, to subtracting 3000 times the divisor from the dividend.

The remainder of the first subtraction being 1142, if we write after it the figures of the dividend, which have not yet been used, there would result a new dividend, upon which we could operate as upon the first dividend; but as we have now to determine the hundreds figure of the quotient, and as the product of the divisor by this figure can-

not give units of a lower order than hundreds, it must be contained wholly in the 11422 hundreds of the remaining dividend; so we bring down to the right of the remainder, 1142, only the following figure 2 of the dividend, which gives a second partial dividend, 11422, upon which we operate as on the first.

The true quotient of 11422 by 2678, is 4, which we write below the divisor, and to the right of the first quotient obtained. We then subtract from the second partial dividend, the product of the divisor by the new quotient. The remainder of this subtraction is 710. We bring down to its right the following figure of the dividend, 9, which gives a third dividend, 7109, and which is to furnish the tens figure of the whole quotient.

Dividing 7109 by 2678, we have for a true quotient 2, which we place at the right of the first two figures of the quotient; multiplying the divisor by 2, and subtracting the product from the third partial dividend, we obtain 1753 for a remainder, to the right of which we bring down the last figure 8 of the dividend, which gives 17538 for a fourth partial dividend. Finally, the true quotient of 17538 by 2678, is 6. We multiply the divisor by 6, and subtract the product from the fourth partial dividend, which gives a remainder, 1470. The required quotient is then 3426, with the remainder 1470, which we can prove by multiplying 2678 by 3426, and adding 1470 to the product. The four operations just performed in this division, conduct to the same result as if we had successively subtracted from the dividend 3000 times, then 400 times, then 20 times, then 6 times the proposed divisor.

Since in division, the dividend is a product of which the divisor and quotient are two factors, it follows that to divide the dividend by a certain entire number, the quotient is by this change divided by the same entire number. For, as, after this change, the quotient multiplied by the divisor must produce a dividend a certain number of times greater or less than the first dividend, it follows necessarily the divisor remaining the same, that the quotient must be the same number of times greater or less.

On the contrary, if, without altering the dividend, we render the divisor a certain number of times greater or smaller, the quotient is thereby rendered the same number of times smaller or greater. Then by dividing the dividend and the divisor by the same number, we do not change the quotient; since, if, by the change of the dividend, we

divide the quotient by a certain number, the second change renders it the same number of times smaller or greater. Thus, the compensation leaves it the same.

MISCELLANEOUS EXAMPLES ON THE INTRODUCTORY PART.

SECTION I.

EXAMPLE 1.—Write in Roman Notation 9, 99, 100, 1000, 2060.

Examples on Notation.

2. Write in figures six, sixty, six hundred, one thousand, nine hundred.

3. Write in figures three trillions, two hundred and forty billions and five.

4. Write in words 35,644,760,027,201.

Addition.

5. Add 500, 650, 4769, 810, 80, 1, and 29 together.

6. Add 2, 20, 35, 595, 606, 890, and 5 together.

Subtraction.

7 From one hundred take ten.

8. From two hundred and fifty take eighty.

9. From one hundred take one.

10. From four hundred and sixty-two take sixty-five.

11. Add 5000, 659, 7005, 89, and 100 together.

12. Add 99, 990, 709, 56, and 47 together.

Multiplication and Division.

13. How many times are 483 contained in 568943?

14. How many times are 6340 contained in 2076907?

15. How many times are 8596 contained in 37695843?

16. Add 55+66+796+94+2 together.

17 Subtract 56785 from 1578790

18. 9340760—65007= ?

19. 725631—4889= ?

20. What is the difference between 1776 and 1867?

21. What is the product of five thousand eight hundred and seventy, by two hundred and sixty-five?

22. What is the product of thirty-nine thousand seven hundred and thirty-six, by eight hundred and twenty-five?

Examples on Addition, Subtraction, Multiplication and Division.

23. Subtract 4596966 from 58568676.
24. How many times are 612 contained in 1000?
25. How many times are 7132 contained in 10003?
26. How many times are 3802 contained in 1000000?
27 Subtract 19656784 from 87613201.
28. 68541—2954 are how many?
29. Twenty thousand times thirty thousand are how many?
30. Sixty-five thousand times five thousand and eighty-four are how many?
31. What is the product of eighty-one thousand two hundred and seven, by three thousand one hundred and forty-five?
32. What is the product of thirty-seven thousand five hundred and sixty-five, by two thousand and fifty-two?
33. From four thousand four hundred and twenty-nine, take two thousand and sixty-eight.
34. From five thousand seven hundred and seventy-seven, take six hundred and eighty-four.
35. Multiply ten millions by ten.
36. Multiply thirty millions by one hundred.
37 Multiply one thousand by one thousand.
38. From one hundred thousand take one.
39. 65+405+769+8505, are how many?
40. 3695÷349, are how many?
41. 78596×695, are how many?
42. 597092—579, are how many?
43. How many times are 5684 contained in 16784220?
44. How many times are 1804 contained in 296784?
45. How many times are 695 contained in 457893?
46. From one thousand take nine hundred and ninety-nine.
47 Multiply seven millions six thousand and thirty by thirty-three.
48. Divide 87658910 by 795.
49. Divide 59374290 by 622.
50. Divide 58427695 by 3652.
51. Multiply 9867849 by 5890

REDUCTION

COMPOUND NUMBERS.

SECTION II.

Reduction defined.

81. *Reduction* is the *change* of numbers from one name or denomination to that of another. but without change of value.

How numbers are changed to different names.

82. When numbers are to be changed from a higher to a lower name, *multiplication* is employed; but when the change is from a lower to a higher one, we use *division*.

Compound numbers alone subject to these changes.

83. Numbers subject to such changes are called *compound*, in distinction to the *simple* numbers already considered.

A compound number defined.

84. A *compound number* is made of two or more unlike denominations; thus dollars, cents, dimes, pounds, shillings and pence, are of this class.

What are not compound numbers.

85. It should be noted, that while the several parts of a compound number, as pounds, shillings, pence, are of different name, they are classed together as relatively alike; but pounds and dollars, and grains and minutes, etc., having no common bond, cannot be reckoned as compound numbers. All of such nature are expressions of unlike values.

How to reduce to a lower name.

86. To reduce a compound number to one of a lower name, observe the following directions:

Multiply the highest denomination in the number to be changed, by that figure which indicates how many ones or units of the next lower denomination are contained in one of the highest, and add to the product the parts of the same value with the multiplier, and so on.

Table of values for memory. Tables for reference.

87. The following tables of values should be thoroughly memorized. The tables of reference beneath them are not to be studied, but used for comparing work done by the pupil. They simply show, in a condensed form, the tables of values, thus:

1 £$=20s.=240d.=960qr.$ or farthings.

ENGLISH MONEY.

88. 4 Farthings, marked *qr* * make 1 penny, *d.* — Table of English money.
12 Pence make 1 shilling, *s.*
20 Shillings make 1 pound, £.
21 Shillings make 1 guinea.

Reference Table.

£		*s.*		*d.*		*qr*
				1	=	4
		1	=	12	=	48
1	=	20	=	240	=	960

EXAMPLE 1.—Reduce £10 15*s.* 8*d.* 2*qr* to farthings.

OPERATION.

```
     20   12   4
10   15    8   2
20
——
215s.
 12
———
2588d.
   4
————
10354qr  Ans.
```

EXPLANATION.

Ten pounds=200*s.*, and the 15*s.* added make 215*s.*; this number multiplied by 12=2580, and with the 8*d.* added, 2588; this multiplied by 4=10352, and the 2 added, 10354, which is the answer. — [illegible] work.

2. Reduce £25 7*s.* 6*d.* 3*qr* to farthings.

Note.—It is convenient to place in small figures, as in the first example, above the numbers of the sum, the several multipliers.

3. Reduce £37 16*s.* 5*d.* to pence. — Examples in English money.
4. Reduce £19 0*s.* $4\frac{3}{4}$*d.* to farthings.
5. Reduce £49 8*s.* $5\frac{1}{4}$*d.* to farthings.
6. Reduce £32 3*s.* 9*d.* to pence.
7. Reduce £50 19*s.* to shillings.
8. Reduce £42 15*s.* 6*d.* to pence.
9. Reduce £513 0*s.* 3*d.* to pence.
10. Reduce £89 1*s.* 2*d.* to pence.
11. Reduce £6 6*s.* 6*d.* to pence.
12. Reduce £48 12*s.* $6\frac{3}{4}$*d.* to farthings.
13. In 5 guineas, how many pence are there?

* Qr. is quarter, the quarter or fourth part of a penny.

14. Reduce 50 guineas to shillings.
15. Reduce 2 guineas to shillings.

Note.—Whenever there is omission of any term, as that of shillings in the 4th sum, it does not affect the multiplication by shillings; thus 19 is to be multiplied by 20, though the order of shillings is not in the sum.

DRY MEASURE.

Dry measure. **89.** 2 Pints, *pt.* make 1 quart, *qt.*
8 Quarts make 1 peck, *pk.*
4 Pecks make 1 bushel, *bush.*
36 Bushels make 1 chaldron, *ch.*

Reference Table.

bush.		*pk.*		*qt.*		*pt.*
				1	=	2
		1	=	8	=	16
1	=	4	=	32	=	64

Note.—By this table, all dry articles, as grain, salt, coal, vegetables, etc., are measured.

EXAMPLE 1.—How many pints are there in 15 bushels, 3 pecks, 5 quarts and 1 pint?

	4	8	2
15 *bush.*	3 *pk.*	5 *qt.*	1 *pt.*
4			
63 *pk.*			
8			
509 *qt.*			
2			
Ans. 1020 *pt.*			

Examples in Dry measure. 2. Reduce 10 bushels, 3 pecks, 5 quarts to pints.
3. Reduce 25 bushels, 2 pecks, 6 quarts to quarts.
4. How many pecks in 35 bushels?
5. How many pints in 28 quarts?
6. How many pints in 3 pecks and 6 quarts?
7. How many quarts in 2 bushels and 2 pecks?
8. Reduce 144 chaldrons to bushels.
9. Reduce 46 chaldrons to pecks.
10. Reduce 25 bushels, 3 pecks, 7 quarts, 1 pint, to pints.

11. Reduce 75 bushels and 2 pecks to quarts.
12. Reduce 250 bushels to pecks.

LIQUID MEASURE.

90. 4 Gills, *gi.* make 1 pint, *pt.*
2 Pints make 1 quart, *qt.*
4 Quarts make 1 gallon, *gal.*
31½ Gallons make 1 barrel, *bbl.*
63 Gallons (31½×2) make 1 hogshead, *hhd.*
2 Hogsheads make 1 pipe, *pi.*
2 Pipes or 4 hogsheads, make 1 tun, *tun.*

The tierce in tables called 42 gallons, is omitted, as it does not represent the tierce used in trade, which has sometimes many more gallons.

Reference Table.

tun.	*pi.*	*hhd.*	*bbl.*	*gal.*	*qt.*	*pt.*	*gi.*
						1=	4
					1=	2=	8
				1 =	4=	8=	32
			1	= 31½=	126=	252=	1008
		1	= 0	= 63 =	252=	504=	2016
	1 =	2 =	0	=126 =	504=	1008=	4032
1 =	2 =	4 =	0	=252 =	1008 =	2016=	8064

By this are measured all liquids, except milk, ale and beer

The Confederate States gallon of liquid measure is 231 cubic inches.

EXAMPLE 1.—In 6 pipes, 3 hogsheads, 15 gallons and 3 quarts, how many quarts?

```
                2          63          4
 6 pi.      3 hhd.     15 gal.     3 qt.
 2
 ---
 15
 63
 ---
 45
90
 15
 ---
960
  4
 ---
Ans. 3843 qts.
```

REMARK.—When a number to be added consists of two figures, as the 15 in this sum, it is more convenient to place as above, than to use it as we do single numbers.

2. Reduce 15 pipes, 1 hogshead, 3 gallons to quarts.
3. Reduce 1 tun, 30 gallons, 2 quarts, 2 pints, 3 gills to gills.
4. Reduce 1 hogshead, 15 gallons, 3 quarts to pints.
5. Reduce 1 barrel, 2 quarts to quarts.

1 *bbl.* = 31½ *gal.* 2 *qt.* 4 —— 124 2 —— 126 2 —— *Ans.* 128 qts.	EXPLANATION. In this example, as the ½ gallon=2 quarts, we simply add ½ of 4 to the quart, one-half gallon being 2 quarts.

6. Reduce 1 hogshead, 1 barrel, 3 quarts, 2 pints to pints.
7. Reduce 1 barrel, 2 quarts to gills.
8. Reduce 1 hogshead, 15 gallons, 3 quarts, 2 pints, 3 gills to gills.
9. In 3 hogsheads, 12 gallons, 3 quarts, how many quarts?
10. In one pipe, 1 hogshead, 5 quarts, how many quarts?
11. In 1 tun, 1 barrel, how many pints?
12. In 2 hogsheads, 14 gallons, how many gallons?

AVOIRDUPOIS WEIGHT.

91. 16 Drachms, *dr.* make 1 ounce, *oz.*
16 Ounces make 1 pound, *lb.*
25 Pounds make 1 quarter, *qr.*
4 Quarters make 1 hundred weight, *cwt.*
20 Hundred weight make 1 ton, *t.*

Reference Table.

t.	*cwt.*	*qr*	*lb.*	*oz.*	*dr*
				1 =	16
			1 =	16 =	256
		1 =	25 =	400 =	6400
	1 =	4 =	100 =	1600 =	25600
1 =	20 =	80 =	2000 =	32000 =	512000

Such articles as sugar, coffee, tea, cotton and metals, with the exception of gold and silver, are weighed by this weight.

In the old tables, 28 lbs. was called a qr., and 112 lbs. a cwt.; but now the standard qr. is 25 lbs., and the cwt. 100 lbs.

EXAMPLE 1.—Reduce 17 tons, 8 hundred weight 3 quarters, 13 pounds, to pounds.

```
        20       4     25
17 t.   8 cwt.   3 qr  13 lb.
 20
----
 348 cwt.
   4
----
1395 qr.
  25
----
6975
2790
  13
-----
Ans.  34888 lbs.
```

2. Reduce 6t. 7cwt. 2qr. 20lb., to oz.
3. Reduce 25t. 5cwt. 1qr. 10lb. 2oz., to oz.
4. Reduce 18lb. 11oz. 12dr., to dr.
5. Reduce 2t. 15oz. 14dr., to dr.
6. Reduce 6qr. 17lb. 13oz. 5dr., to dr.
7. Reduce 3t. 22 lb., to oz.
8. Reduce 3qr. 15oz., to dr.
9. In 3cwt. 20lb., how many pounds?
10. In 9qr 12lb. 5oz., how many ounces?

APOTHECARIES WEIGHT.

92. 20 Grains, *gr.* make one scruple, ℈.
3 Scruples make 1 drachm, ʒ.
8 Drachms make 1 ounce, ℥.
12 Ounces make 1 pound, ℔.

Reference Table.

lb.		*oz.*		*dr*		*sc*		*g.*
						1	=	20
				1	=	3	=	60
		1	=	8	=	24	=	480
1	=	12	=	96	=	288	=	5760

Medicines are compounded by this weight, but bought and sold by Avoirdupois weight.

There is no difference in the pound, ounce and grain of this and Troy weight, but the ounce is differently subdivided.

EXAMPLE 1.—Reduce 6 pounds, 7 ounces, 5 drachms, 2 scruples, 12 grains, to grains.

	12	8	3	20
6℔	7℥	5ʒ	2℈	12gr.
12				
79				
8				
637				
3				
1913				
20				
Ans. 38272 *gr.*				

2. Reduce 25℔. 8oz. 2dr. 15sc., to scruples.
3. Reduce 80℔. 9oz. 1dr., to drachms.
4. Reduce 2℔. 3oz. 2dr., to scruples.
5. In 30℔. 8oz. 2dr. 12sc., how many scruples?
6. In 9oz. 5dr., how many drachms?
7. In 50℔. 4oz. 1dr., how many scruples?
8. In 12℔. 2dr., how many scruples?
9. In 29℔. 6oz. 1dr. 16sc., how many grains?
10. In 5oz. 2dr. 15sc., how many grains?

TROY WEIGHT.

Troy weight. **93.** 24 Grains, gr. make 1 pennyweight, *pwt.*
20 Pennyweights make 1 ounce, *oz.*
12 Ounces make 1 pound, *lb.*

Reference Table.

lb.		*oz.*		*pwt.*		*gr.*
				1	=	24
		1	=	20.	=	480
1	=	12	=	240	=	5760

This is the standard measure for gold, silver, jewels, corn, bread and liquors.

Example 1.—Reduce 15 pounds, 9 ounces, 14 penny weights, 12 grains, to grains.

```
            12      20        24
     15 lb.  9 oz.  14 pwt.  12 gr
     12
   -----
    189 oz.
     20
   -----
   3794 pwt.
     24
   -----
  15176
  3588
     12
   -----
Ans. 51068 gr.
```

2. Reduce 21lb. 9oz. 14pwt. 12gr., to grains.
3. Reduce 5oz. 14pwt., to pennyweights.
4. Reduce 55lb. 10oz., to grains.
5. Reduce 9oz. 15pwt. 15gr., to grains.
6 Reduce 16pwt. 6gr., to grains.
7. Reduce 10lb. 6oz., to pennyweights.
8. In 13lb. 16pwt. 15gr., how many grains?
9. In 35lb. 14pwt. 10gr., how many grains?
10. In 60lb. how many pennyweights?

ALE OR BEER MEASURE.

94. 2 Pints, *pt.* make 1 quart, *qt.* Ale or beer measure.
4 Quarts make 1 gallon, *gal.*
36 Gallons make 1 barrel, *bar.*
54 Gallons make 1 hogshead, *hhd.*

Reference Table.

hhd.		*bar.*		*gal.*		*qt.*		*pt.*
						1	=	2
				1	=	4	=	8
		1	=	36	=	144	=	288
1	=	$1\frac{1}{2}$	=	54	=	216	=	432

1 Gallon contains 282 cubic inches.

EXAMPLE 1.—Reduce 25 hogsheads, 3 quarts, 2 pints, to pints.

```
             4     2
   25hhd.  3qt.  2pt.
   54
   ——
  100
  125
  ———
 1350 gal.
    4
 ————
 5403 qt.
    2
 —————
Ans. 10808 pt.
```

Note.—Though no gallons are named in the sum, we still have to multiply by the number of gallons that make a hogshead.

2. Reduce 5hhd. 2bar. 3qt. 1pt., to pints.
3. Reduce 30hhd. 3bar. 2pt., to pints.
4. Reduce 15hhd. 15gal. 3 qt., to quarts.
5. Reduce 12hhd. 20gal. 2qt., to pints.
6. Reduce 25gal., to pints.
7. Reduce 1bar. 3qt., to pints.
8. In 8hhd., how many quarts?

CLOTH MEASURE.

Cloth measure.

95. $2\frac{1}{4}$* Inches, *in.* make 1 nail, *na.*
4 Nails make 1 quarter, *qr.*
4 Quarters make 1 yard, *yd.*
3 Quarters make 1 Ell Flemish, *E. Fl.*
5 Quarters make 1 Ell English, *E. Eng.*

Reference Table.

E. Eng.		*yd.*		*E. Fl.*		*qr.*		*na.*		*in.*
								1	=	$2\frac{1}{4}$
						1	=	4	=	9
				1	=	3	=	12	=	27
		1	=	$1\frac{1}{3}$	=	4	=	16	=	36
1	=	$1\frac{1}{4}$	=	$1\frac{2}{3}$	=	5	=	20	=	45

Cloths, carpets and all articles of the yard measurement are sold by this.

* The fractional $\frac{1}{4}$, and other like expressions in the Tables, will be explained under the head of fractions.

EXAMPLE 1.—Reduce 36 yards, 3 quarters, 2 nails, to nails.

	4	4
36yd.	3qr.	2na.
4		
147 qr.		
4		
Ans. 590 na.		

2. Reduce 25yd. 2qr., to nails.
3. Reduce 350yd. 3qr. 3na., to nails.
4 Reduce 3qr. 3na., to nails.
5. Reduce 12yd., to quarters.
6. In 28E. Fl., how many nails?
7 In 17E. Eng., how many quarters?
8. In 60E. Fl., how many yards?
9. In 37E. Eng., how many yards?
10. In 45yd. 3qr., how many quarters?

LONG MEASURE.

96. 3 Barleycorns, *b.c.* make 1 inch, *in.* Long measure

12 Inches make 1 foot, *ft.*
3 Feet make 1 yard, *yd.*
5½ Yards, or (5½×3) 16½ feet, make 1 rod, *rd.*
40 Rods make 1 furlong, *fur.*
8 Furlongs make 1 mile, *mi.*
3 Miles make 1 league, *l.*
69½ Statute miles, nearly, or 60 geographical miles, make 1 degree or circumference of the earth, *deg* or °
360 Degrees make 1 circumference, *circ.*

Reference Table.

mi.	*fur.*	*rd.*	*yd.*	*ft.*	*in.*	*b.c.*
					1 =	3
				1 =	12 =	36
			1 =	3 =	36 =	108
		1 =	5½=	16½=	198 =	594
	1 =	40 =	220 =	660 =	7920 =	23760
1 =	8 =	320 =	1760 =	5280 =	63360 =	190080

By this is measured distances, lengths, breadths, etc

A fathom is a length of 6 feet, and is used, principally, for soundings, at sea. A fathom

A hand. A hand is 4 inches, and is used to find the height of horses.

To multiply by $\frac{1}{2}$. To multiply by $\frac{1}{2}$, as in the Rod measure, we simply take one-half of the number to be multiplied, and add; thus $8 \times 5\frac{1}{2} = 40 \times 4$ (or the half of 8) $= 44$.

EXAMPLE 1.—Reduce 4 rods, 2 feet, 8 inches, to barleycorns.

3 12 4 rd. 2 ft. 8 in. $5\frac{1}{2}$ $\overline{20}$ 2 $\overline{22}$ yd. 3 $\overline{68}$ ft. 12 $\overline{824}$ in. 3 *Ans.* $\overline{2472}$ b.c.	Here it will be seen, that although not named in the sum, the rods have, first, to be changed to yards before they can be reduced to feet. The 2 added to the 20 is the $\frac{1}{2}$ of 4 rods.

2. Reduce 10mi. 6fur. 25rd., to feet.
3. Reduce 25° 30mi. 5fur. 10rd. 3yd. 2ft., to inches.
4. Reduce 50mi. 6fur. 8rd. 5yd., to inches.
5. Reduce 5mi. 5rd. 5ft., to feet.
6. In 95° how many miles?
7 In 5fur. 6rd. 9yd. 7in., how many inches?
8. In 8rd. 9yd. 3qr. 2ft., how many feet?

LAND OR SQUARE MEASURE.*

Square measure. **97.** 144 Square inches, *sq. in.*, make 1 square foot, *sq. ft*

9 Square feet make 1 square yard, *sq. yd.*

$30\frac{1}{4}$ Square yards, or $272\frac{1}{4}$ square feet make 1 square rod, *sq. rd.*

40 Square rods make 1 rood, *r.*

4 Roods make 1 acre, *a.*

640 Acres† make 1 square mile, *sq. mi.*

A square defined. * A square is a figure bounded by four equal lines at right angles to each other. Each line is known as a side of the square.

† An acre contains 4840 square yards. For larger areas. there is the square, one of the sides of which is a mile. The square mile, containing 640 acres, is called a section in the classification of public lands.

Reference Table

mi.	*a.*	*r.*	*sq. rd.*	*sq. yd.*	*sq. ft.*	*sq. in.*
					1 =	144
				1 =	9 =	1296
			1=	30¼=	272¼=	39204
		1 =	40=	1210 =	10890 =	1568160
	1=	4=	160=	4840 =	43560 =	6272640
1=	640=	2560=	102400=	3097600 =	27878400 =	4014489600

Surfaces are measured by this.

A square yard contains 9 square feet; the product of .3. A square foot contains 144 square inches, the oduct of 12×12.

The multiplication by the fractional figure ¼ in the are rod, is performed by taking one-fourth of the mber to be multiplied, and adding it; thus, 40×30¼= 00×10=1210. Multiplication of the fraction ¼.

EXAMPLE 1.—Reduce 1 square mile, 2 acres, 2 roods, to uare yards.

```
      640    4
1 sq. mi.  2 a.  2 r.
 640
 642 a.
   4
2570 r.
  40
102800 sq. r.
    30¼
3084000
  25700
is. 3109700 sq. yd.
```

It is the same to take ¼ of a number, as to multiply by 1 and then divide by 4; thus, 4)102800 gives 25700

Reduce 10a. 5r. 3sq. yd., to square feet.
Reduce 50a. 3r. 6sq. ft., to square inches.
Reduce 25a. 14sq. r., to square feet.
Reduce 45r. 8sq. yd., to square inches.
In 3sq. mi., how many square feet?
7In 2a 2r., how many square feet?
8In 3sq. yd. 2sq. ft., how many square inches?

CIRCULAR AND ASTRONOMICAL MEASURE.

9 60 Seconds, 60″ make 1 minute, 1′
60 Minutes make 1 degree, 1°
30 Degrees make 1 sign, s. Circular and astronomical measure
12 Signs, or 360° make 1 circle, *circ*

Reference Table.

circ.		*s.*		°		′		60″
				1	=	1	=	3600
		1	=	30	=	60	=	108000
1	=	12	=	360	=	1800	=	1296000

This measure is used to calculate latitude and longitude and astronomical distances.

Divisions of circles, large or small.

Circles of all sizes are supposed to have the divisions of 360 equal parts, called degrees; these degrees are divided into 60 equal parts called minutes; and these minutes, each, into 60 seconds.

EXAMPLE 1.—Reduce 4 signs, 25°, 15′, to minutes.

```
        30     60
  4s.   25°    15′
  30
 ----
 120
  25
 ----
  45°
   60
 -----
 8700
   15′
 -----
Ans. 8715′
```

2. Reduce 5circ. 2s. 3°, to minutes.
3. Reduce 4circ. 15° 16′, to seconds.
4 In 8circ. 10° 14′, how many seconds?
5. In 10° 15′, how many seconds?
6. In 6circ. 9s. 14°, how many minutes?

SURVEYOR'S MEASURE.

Surveyor's measure.

99. $7\frac{92}{100}$ Inches, *in.* make 1 link, *li.*
25 Links make 1 rod, perch or pole, *rd.*
4 Rods make 1 chain, *ch.*
10 Chains make 1 furlong, *fur.*
8 Furlongs make 1 mile, *mi.*

Also,

10,000 Square links, or 16 square rods ma square chain, *sq. ch.*
10 Square chains make 1 acre, *a.*

Reference Table.

mi.		*fur.*		*ch.*		*rd.*		*li.*		*in.*
								1	=	$7\frac{92}{100}$
						1	=	25	=	198
				1	=	4	=	100	=	792
		1	=	10	=	40	=	1000	=	7920
1	=	8	=	80	=	320	=	8000	=	63360

This is used in laying out railroads, and measuring the boundaries of fields.

The surveyor's chain is 4 poles or 66 feet long; it is divided into 100 links.

EXAMPLE 1.—Reduce 25 miles, 5 furlongs, 3 chains, 10 rods, to links.

```
             8       10       4
  25 mi.   5 fur.   3 ch.   10 rd.
   8
  ---
  205 fur.
   10
  ----
  2053 ch.
     4
  ----
  8212
    10
  ----
  8222 rd.
    25
  -----
  41110
 16444
 ------
Ans. 205550 li.
```

2. Reduce 8fur. 6ch. 9rd., to rods
3. Reduce 30mi. 3ch. 8rd., to links.
4. Reduce 9ch. 3rd., to links.
5. Reduce 5mi., to rods.
6. In 15 mi. 6fur. 3ch. 2 rd., how many rods?
7. In 7ch. 3rd., how many links?
8. In 30fur. 9ch. 5rd., how many rods?

SOLID OR CUBIC MEASURE.

100. 1728 Cubic inches, *cu. in.* make 1 cubic foot, *c. ft.* Solid or cubic measure.
27 Cubic feet make 1 cubic yard, *c. yd.*

40 Cubic feet make 1 ton of timber, *t.*
16 Cubic feet make 1 cord foot, *c. ft.*
8 Cord feet, or 128 cubic feet make 1 cord, *c.*

Reference Table.

cu. yd.		*cu. ft.*		*cu. in.*
		1	=	1728
1	=	27	=	46656

This is used to measure what has length, breadth and thickness.

A cord measure. A cord of wood is 4 feet wide, 4 feet thick and 8 feet long.

A cube defined. A cube is a figure of six equal squares, called faces; the sides of the squares are called edges. The face on which a cube stands is called its base. If the edge is one yard, it will contain $3\times3=9$ square feet; therefore, 9 cubic feet can be placed on the base; and hence if the figure were 1 foot thick, it would contain 9 cubic feet; if it were 2, it would contain twice as many; if 3, 27 feet.

To find the contents of a cube. The contents of a cube are found by multiplying together the length, breadth and thickness.

The loss in hewing timber. Round timber is estimated to lose one-fifth by squaring.

EXAMPLE 1.—Reduce 18 cubic yards, 18 cubic feet, 15 cubic inches, to inches.

```
               27          1728
  18 cu. yd.   18 cu. ft.  15 cu. in.
  27
 ----
 126
 18
  18
 ----
 324 cu. ft.
1728
 ----
2592
 648
2268
324
------
Ans. 559872 cu. in.
```

2. Reduce 1c. 5cu. yd. 20cu. ft., to inches.
3. Reduce 16cu. yd. 15cu. ft., to feet.
4. Reduce 12cu. yd. 10cu. ft., to inches.
5. Reduce 5c., to cubic inches.
6. In 25 cords of wood, how many cord feet?
7. In 30 cords of wood, how many cubic feet?
8. In 120 feet round timber, how many inches?

TIME MEASURE.

101. 60 Seconds, *sec.* make 1 minute, *m.* — Time measure.
60 Minutes make 1 hour, *h.*
24 hours make 1 day, *d.*
7 Days make 1 week, *wk.*
4 Weeks make 1 lunar month, *l. m.*
12 Months make 1 calendar year, *c. yr.*
13 Months, 1 day and 6 hours make 1 Julian year, *J. yr.*

Reference Table.

J. yr.	*l. m.*	*wk.*	*d.*	*h.*	*m.*	*sec.*
					1 =	60
				1 =	60 =	3600
			1 =	24 =	1440 =	86400
		1 =	7 =	168 =	10080 =	604800
	1 =	4 =	28 =	672 =	40320 =	2419200
1 =	$13\frac{5}{112}$ =	$52\frac{5}{28}$ =	$365\frac{1}{4}$ =	8766 =	525960 =	31557600

Note.—As the length of the year is 365 days and 6 hours, the odd hours in 4 years make 1 day, which is added to every fourth year, in the month of February. The year, thus increased, is called Leap Year. — Leap year.

Years exactly divisible by 4, as 1860, 1864, 1868, are leap years.

The following verse memorized, is of use to recall the number of days in each month: — The days of the month.

Thirty days hath September,
April, June and November;
All the rest have thirty-one
Except the second month alone,
And that has eight and twenty, clear,
But nine and twenty each Leap Year

EXAMPLE 1.—Reduce 5 years, 9 months, 6 days, 12 hours, 15 minutes, to minutes.

	12	7	24	60
5 yr.	9 mo.	6 da.	12 hr.	15 m.

12
——
69 mo.
4
——
276 wk.
7
——
1938 da.
24
——
7752
3876
12
——
46524 hr.
60
——
2791440
15
——
Ans. 2791455 m.

In this sum let it be noted that no weeks are named.

102. 2. Reduce 1J. yr. to minutes.
3. Reduce 5mo. 5da. 16hr., to minutes.
4. In 25yr. 2wk. 5da., how many hours?
5. In 1da. 18hr. 20m., how many seconds?
6. In 15hr. 35m. 40sec., how many seconds?
7 Reduce 1c. yr. to minutes.
8. Reduce 3mo. 5wk. 5da., to hours.
9. Reduce 2J. yr. to minutes.
10. Reduce 3c. yr. to hours.

MISCELLANEOUS TABLES.

103. 12 Units, or things, make 1 dozen.
12 Dozen make 1 gross.
12 Gross, or 144 dozen, make 1 great gross.
20 Units, or things, make 1 score.
196 Pounds make 1 barrel of flour.
100 Pounds make 1 quintal of fish.
200 Pounds make 1 barrel of pork.
18 Inches make 1 cubit.
14 Pounds of iron or lead make 1 stone.
21½ Stones make 1 pig.
8 Pigs make 1 fother.
24 Sheets of paper make 1 quire.
20 Quires make 1 ream.

104. FOREIGN COINS.

COUNTRY.	GOLD COINS.		SILVER COINS.	
	DENOMINATION.	VALUE.	DENOMINATION.	VALUE.
		$ cts. m.		$ cts. m.
Austria,	Ducat,	2 28 0	Scudo,	1 01 5
Belgium,	25 Francs,	4 72 0	5 Francs,	96 8
Bolivia,	Doubloon,	15 58 0	Dollar,	1 05 4
Brazil,	20,000 Reis,	10 90 5	2000 Reis,	1 01 3
Chili,	10 Pesos,	9 15 3	New Dollar,	97 0
Denmark,	10 Thaler,	7 90 0	2 Rigsdaler,	1 09 4
England,	Sovereign, new,	4 86 3	Shilling, new,	22 7
England,	Sovereign, average,	4 84 8	Shilling, average,	22 2
France,	20 Francs, average,	3 84 5	5 Francs, average,	95 8
Germany, north,	10 Thaler,	7 90 0	Thaler,	71 7
Germany, south,	Ducat,	2 28 3	Guilder or Florin,	41 2
Mexico,	Doubloon, average,	15 53 4	Dollar, average,	1 04 9
Netherlands,	10 Guilders,	3 99 0	2½ Guilders,	1 02 3
New Granada,	10 Pesos, new,	9 67 5	Dollar, 1857,	96 8
Peru,	Doubloon, old,	15 56 0	Dollar, 1855,	93 6
Portugal,	Crown,	5 81 3	Crown,	1 16 6
Rome,	2½ Scudi, new,	2 60 0	Scudo,	1 04 7
Russia,	5 Roubles,	3 97 6	Rouble,	78 4
Spain,	100 Reals,	4 96 3	Pistareen, new,	20 1
Sweden,	Ducat,	2 26 7	Rix dollar,	1 10 1
Turkey,	100 Piastres,	4 37 4	20 Piastres,	86 5
Tuscany,	Sequin,	2 30 0	Florin,	27 4

Note.—The above values are computed at the Mint rate of $18.60 per ounce standard (9-10 fine) for gold, and $1.21 per ounce standard for silver.

Note.—The English pound, or pound sterling, is valued at $4.44c. 4m. (Art. 285). The French franc is valued at 18¾ cents (Art. 286).

RULES FOR MEASURING CRIBS, HOGSHEADS, ETC.

To find the number of cubic feet in any square crib or box, multiply the length by the breadth (in feet) for the number of square feet on the floor, and this product by the depth, for the required number of cubic feet in the box or room. Thus if a room be 12 feet long by 6 wide, it contains $12\times6=72$ square feet on the floor, and if 5 feet deep, it contains $72\times5=360$ cubic feet.

To find the number of bushels: A cubic foot contains 1728 cubic inches—and a bushel about 2160 (accurately, 2150.42) inches. A cubic foot is therefore 1728–2160= 4–5 or 8–10 of a bushel. A wine gallon contains 231 cubic inches. A cubic foot therefore contains about 7 1–2 and a bushel about 9 1–3 wine gallons.

Corn is usually put up on the cob or in the shuck, while it is sold by the bushel or barrel of shelled corn. The proportion of shelled corn to corn on the cob is nearly uniform, but compared with corn in the shuck it varies considerably—depending on—1, the size of the ears—2, the way it is shucked, and—3, the way it is packed or trodden in. One bushel of shelled corn is equal to two bushels of corn on the cob, to about three bushels of corn in slip shuck (say 2½ to 3½), and to about 4 of corn in full shuck (say 4 to 4½).

If a crib of corn on the cob is 12 feet long, 10 wide, and 8 deep, it will hold as follows:

12	Length in feet.
10	Width.
120	Square feet on floor.
8	Depth.
960	Cubic feet,
8	=8–10 Multiplier for bushels.
7680	(The right hand figure cut off) number of bushels of corn on the cob—768.
2—7680	
384	Number of bushels of shelled corn.
3—768	Bushels—if in slip shuck.
256	Bushels of shelled corn.
4—768	Bushels—if in full or whole shuck.

192 Bushels of shelled corn.
5—384 Bushels of shelled corn.

——

76 4–5 Barrels of shelled corn.

Note.—If the corn be not level, it must be made so or averaged.

A concise rule for finding the contents, in shelled corn, of a crib of corn put up in the cob.

Multiply together the length, breadth and average depth, expressed in feet. Multiply this product by 4, and cut off one figure from the right, for the answer in bushels of shelled corn.

EXAMPLE.

In a crib 15 feet long, 12 feet wide, filled 9 feet deep with corn on the cob, how many bushels of shelled corn?

15 Length in feet.
12 Width.

——

180 Square feet on floor.
9 Depth.

——

1620 Cubic feet.
4 Multiplied for bushels.

——

648,0 (One decimal cut off) 684 bushels shelled corn.

If the corn be in slip shuck, multiply the cubic feet by 3, and if in full shuck, by 2, and cut off one figure as decimal for the answer in bushels of shelled corn.

A concise rule for reducing corn on the cob, to barrels, of shelled corn:

Take 8 per cent. of the product of length, width and depth, expressed in feet.

EXAMPLE.

In a crib of corn on the cob 20 feet long, 10 wide, and 9 deep, how many barrels of shelled corn?

20×10=200—200×9=1800
8 per cent

——

14400 cut off 2 decimals=144 bbls.

For all grain, wheat, shelled corn, etc., which are sold as they stand, multiply together the length and breadth and depth in feet for the number of cubic feet; multiply

this by 8 and cut off one decimal for the answer in bushels.

EXAMPLE.

A box of wheat is 12 feet long, 4 wide and 5 deep: how many bushels does it contain?

$12 \times 4 = 48 — 48 \times 5 = 240$

```
                240
                  8
               ----
              192,0—192 bushels.
```

RULE FOR PEAS IN THE SHELL.

Multiply together the length and breadth and depth in feet for the number of cubic feet; divide this product by 20 for the number of bushels of shelled peas.

EXAMPLE.

In a room, of unshelled peas, 20 feet long, 15 wide, and averaging 6 feet deep, how many bushels of shelled peas?

$20 \times 15 = 300.—300 \times 6 = 1800.—1800—20 = 90$ bushels.

To find the number of bushels in a hogshead, barrel, or other vessel of a circular base, and approximating a cylinder in form, measure the inside diameter one-third of the way down from the top, and the depth, in inches

RULE.

Multiply the diameter in inches by itself, and the product by the depth. Then multiply by 36½, and cut of 5 decimals for the answer in bushels.

EXAMPLE.

In a hogshead whose depth is 40 inches, and the diameter (one-third from the top) 30 inches, how many bushels?

```
    30   diameter in inches.
    30
   ---
   900
    40  depth.
 -----
 36000
   36½
 -----
 216000
108000
  18000
-------
1314000 (5 decimals)—13 bushels, 14-000.
```

To find the number of bushels in a potato bank, piled in the form of a cone:

RULE.

Multiply the diameter at the base by itself, and the product by the height in feet. Then multiply by 21 and cut off 2 figures for decimals for the answer in bushels.

EXAMPLE.

In a potato bank, the diameter being 6 feet at the base, and the height 5 feet, how many bushels?

$6\times6=36$—$36\times5=180$

```
   180
    21
  ----
   180
  360
  ----
 3,780=37 8-10 bushels.
```

If the potatoes do not come to a point at the top, but round considerably; then divide the 180 by 4 for the answer—say 180—4=45 bushels.

TO MEASURE BY A MEASURING ROD.

Cut a rod exactly 51⅝ inches long, and measure it off into 4 equal parts. Each part will be a line * bushel. A box just as long, wide and deep as this would contain exactly one bushel. Subdivide each line bushel into ten equal parts calling them tenths.

When the dimensions are found with this rod, the product of length, breadth and depth is the answer in bushels.

EXAMPLE.

A crib is 10 line bushels long, 8 wide, and 5 4-10 (or 5-4) deep: how many bushels does it contain?

```
  10  line bushels long.
   8  wide.
  --
  80
  54
 ---
 320
400
----
432,0  number of bushels—432.
```

* 12 906-1000 inches.

If the crib is full of corn on the cob, divide by 2 to reduce it to shelled corn, and so in other cases.

EXAMPLE 1.—How much clear corn in a bin 5 feet high, 8 wide and 16 long, the corn being in full shuck?

2. How much clear corn in a bin 12 feet wide, 10 high and 20 long, the ears being in slip shuck?

3. What is the quantity of corn in a bin $6\frac{1}{2}$ feet wide, $8\frac{1}{2}$ feet high and $10\frac{1}{2}$ feet long, the ears being without shuck?

4. How many bushels of peas in a room 20 feet wide, 30 long and 42 high?

5. How many bushels of potatoes in a bank, the diameter at the base being 8 feet and the height 7 feet?

6. A crib is 9 line bushels long, 7 wide and 3 3-10 (or 3, 3) deep: how many bushels?

Changing from a lower to a higher name.

105. To change numbers from a lower to a higher name, without change of value, we employ division. This is a simple reverse of the process, used in reducing from a higher to a lower denomination.

EXAMPLE 1. Reduce 10354 farthings to £.*

Explanation of work.

OPERATION.

```
      4)10354
      -------
     12)2588 2qr.
      -------
      20)215 8d.
      -------
Ans.    £10 15s. 8d. 2qr.
```

EXPLANATION.

We first divide by 4, since there can be only one-fourth as many pence as farthings. By this division are found 2588d. and a remainder of 2qr. To reduce these pence to shillings, we divide by 12, since there can be only one-twelfth as many shillings as pence, and we get 215s. and also have a remainder of 8d. To reduce these shillings to pounds, we divide by 20, and find for result, £10 and a remainder of 15s. Placing by the side of this figure, £10, the several remainders in their proper order, we find that 10354 farthings are, when reduced, £10 15s. 8d. 2qr.

106. In a similar manner, like examples are to be performed according to the following directions:

How to reduce from a lower to a higher name.

Divide the sum directed to be reduced by the number of its denomination that makes it higher; divide that result by the number of its denomination, and so on. The final quotient and the several remainders are what were to be found.

---

* Ex. 1., Art. 88. shows the reverse process.

107. It will be seen by the comparison of Ex. 1, Art. 88, with Ex. 1, Art. 99, that the two kinds of reduction prove each other.

The two kinds prove each other.

EXAMPLE 1.—Reduce 1365 inches to rods.

OPERATION.

12)1365
———
3)113 ft. 9 in.
———
$5\frac{1}{2}$)37 yds. 2 ft.
———
11)74
———

Ans. 6 rods, 4 yd. 2 ft. 9 in.

Note.—In this example it will be seen that the division by $5\frac{1}{2}$ is easily performed by doubling that number, and the dividend, 37, so that they stand 11)74. Such change never affects the value of a term, while it relieves from the embarrassment of fractional division. A similar method is to be observed in dividing by any number connected with $\frac{1}{4}$, $\frac{1}{3}$, etc., only changing the divisor and dividend into fourths, thirds, etc., as the case requires.

Dividing by $5\frac{1}{2}$ or similar expressions.

2. Reduce 455 pints, Dry measure, to higher denominations.
3. Reduce 29795 cubic inches to feet and yards.
4. Reduce 177564 farthings to £.
5. Reduce 65432 shillings to £.
6. Reduce 59678 pence to shillings.
7. Reduce 965430 pence to £.
8. Reduce 3764354 pounds to tons.
9. Reduce 545509 grains, Apothecaries' weight, to pounds.
10. In 2500 nails, how many yards?
11. Reduce 5665 rods to miles.
12. Reduce 3567 links to miles.
13. In 9657840 cu. in., how many cords?
14. Reduce 985067 sq. r. to sq. miles.
15. What number of circles in 1296000 seconds?
16. How many gallons in 835 gills?
17. In 896574 seconds, how many calendar months?
18. In 765325 ounces, how many tons?
19. In 57850 links, how many miles?

MISCELLANEOUS EXAMPLES.

108. Example 1.—Reduce £75 15s. 9d., to pence.
2. Reduce £5 6s. 7d. 3qr., to farthings.
3. Reduce 6078095 farthings to £.
4. Reduce 695432 pence to £.
5. Reduce 6bush. 2p. 6qt. 1pt., to pints.
6. Reduce 50t. 15cwt. 2qr. 16lb. 12oz. 8dr., to drachms.
7 Reduce 3c. yd. 20c. ft. 1435c. in., to cubic inches.
8. Reduce 5mi. 5fur. 6ch. 2rd. 16li., to links.
9. Reduce 14lb. 9℥. 5ʒ. 2℈. 16gr., to grains.
10. Reduce 65lb. 9oz. 15pwt. 18gr., to grains.
11. Reduce 5yd. 2qr. 3na., to nails.
12. Reduce 578965 links to miles.
13. Reduce 694205c. in. to cubic yards.
14. Reduce 787650na. to yards.
15. Reduce 3yd. 2ft. 6in. 2b. c., to barley corns.
16. Reduce 4m. 4fur. 25rd., to rods.
17. Reduce 26sq. m. 30a. 2rd. 33sq. rd., to square rods.
18. Reduce 9gal. 3qt. 2pt. 3gi., to gills.
19. Reduce 6378 gills to gallons.
20. Reduce 12circ. 6s. 15° 45′ 10″, to seconds.
21. Reduce 5mo.3wk. 16h. 27sec., to seconds.
22. Reduce 3m. 6ch. 2rd., to links.
23. Reduce 3t. 18lb., to ounces.
24. In 75a. 6r. 5sq. rd., how many square inches?
25. In 30mo. 3w. 6da. 15h., how many minutes?

AMERICAN MONEY

SECTION III.

109. *American Money* is the currency of the Southern Confederacy. Currency of the Confederate States.

110. It is known by the several names of eagles, dollars, dimes, cents and mills. Its divisions.

111. The gold coin is the eagle, half-eagle, quarter-eagle and dollar. Gold coins.

112. The silver coin is the dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent piece. Silver coins.

113. The nickel is the cent, commercially called the copper. Nickel, formerly a copper coin.

AMERICAN MONEY TABLE.

114. Table of American currency.

10 mills make 1 cent, marked *ct.*
10 cents make 1 dime, marked *d.*
10 dimes make 1 dollar, marked $
10 dollars make 1 eagle, marked *E.*

Reference Table.

Eagle.		*Dollars.*		*Dimes.*		*Cents.*		*Mills.*
						1	=	10
				1	=	10	=	100
		1	=	10	=	100	=	1000
1	=	10	=	1000	=	100	=	10,000

115. As this money comes under the directions for performing addition, subtraction, multiplication and division, the general subject of this currency is referred to them. The directions to its use.

116. A point is used to separate dollars from cents; thus $5.66 is read 5 dollars and sixty-six cents; without the point, $566 would be five hundred and sixty-six dollars. The point distinguishing the $ and cents.

117. When three figures are at the right of the point, the two first are cents, and the third figure is mills; thus, $5.666 expresses five dollars, sixty-six cents, six mills. If a comma had been where the point stands, it would be read five thousand six hundred and sixty-six dollars. The figures expressing cents and mills.

How parts of a dollar are expressed.

118. Parts of a dollar are often expressed in numbers, known as fractions, thus:

50	cents, or a half-dollar, is written - - -	$\frac{1}{2}$
$33\frac{1}{3}$	cents, or a third-dollar, is written -	$\frac{1}{3}$
25	cts., or one-fourth or one-quarter dollar, is written	$\frac{1}{4}$
20	cents, or one-fifth dollar, is written -	$\frac{1}{5}$
$12\frac{1}{2}$	cents, or one-eighth dollar, is written -	$\frac{1}{8}$
10	cents, or one-tenth dollar, is written -	$\frac{1}{10}$
$6\frac{1}{4}$	cents, or one-sixteenth dollar, is written	$\frac{1}{16}$
5	cents, or one-twentieth dollar, is written -	$\frac{1}{20}$
5	mills, $\frac{1}{2}$ of a cent.	

Cents changed to mills; dollars to cents and mills. The eagle changed to dollars. Examples of change.

119. Cents become mills by the annexation of one cipher; dollars become cents by the annexation of two ciphers, and mills by three; and eagles become dollars by the annexation of one cipher; thus, 60 cents are 600 mills; 65 dollars are 6500 cents, 65,000 mills; 75 eagles are 750 dollars.

EXAMPLE 1.—Write 45 dollars, 46 cents, 6 mills, in mills.

OPERATION.

45 dollars=	4500	cents.
Add to these	46	cents.
	4546	cents.
Annex one cipher to change to mills,	45460	mills.
Add	6	mills.
Ans.	45,466	mills.

2. In 35 dollars, 63 cents, 5 mills, how many mills?
3. In \$65, how many cents?
4. In \$550, how many cents? how many mills?
5. In \$5.60, how many mills?
6. In \$2.50, how many cents?
7. In \$100, how many cents?
8. In 5 E and \$6.40, how many cents?

Mills changed to cents; to dollars.

120. To change mills to cents, the right hand figure must be cut off; and mills to dollars, the three right hand figures.

Cents changed to dollars.

121. To change cents to dollars, the two right hand figures must be cut off.

REMARK.—When dollars are multiplied by dollars, the answer is in dollars; when by cents, the answer is in cents; and when cents are multiplied by cents, the answer is in mills.

9. Change 36445 mills to dollars, cents and mills.
Ans. 36,45,5, or \$36.45c. 5m.
10. Change 6954320 mills to dollars, cents and mills.
11. Change 78654461 mills to dollars and cents.
12. Change 5567905 mills to dollars and cents.
13. Change 3890076 mills to dollars, cents and mills.
14. Change 3650123 mills to dollars and cents.
15. Change 984060 mills to dollars, cents and mills.

122. The mill is simply an imaginary coin, and in commercial transactions hardly known; thus, in the sale of articles, amounting severally to — The mill an imaginary coin.

	$62\frac{1}{2}$
	$43\frac{3}{4}$
	$1.31\frac{1}{4}$
Which added, are	\$$2.37\frac{1}{2}$

The trader does not express the $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ in mills, but adds them as fractions, and writes the same as above, not as the amount equally is, \$2.37c. 5m.

REMARK.—The pupil must be particular in putting the separation point between the dollars and cents; as also, when adding, to place cents under cents, and dollars under dollars. — The separation point not to be neglected; also, the arrangement of similar values.

123. When one figure only expresses the cent, a cipher is to be placed at its left; thus to write four dollars and six cents, we do not write \$4.6, but \$4.06. — How to write in figures the cent.

124. In the following examples, in the *Addition of American Money*, the answers can be found by the table, or in the commercial form of adding fractions. — Addition of American currency.

EXAMPLE 1.—John bought 6 pair socks for \$1.25, a vest for \$2.25, a coat for \$9 $37\frac{1}{2}$, 6 handkerchiefs for \$$1.62\frac{1}{2}$, and a cravat for 75c : what was the cost ?

OPERATION.

Socks,	\$1.25	=	\$1.25	or	\$1.25
Vest,	2.25	=	2.25		2.25
Coat,	$9.37\frac{1}{2}$	=	9.375		$9.37\frac{1}{2}$
Handkerchiefs,	$1.62\frac{1}{2}$	=	1.625		$1.62\frac{1}{2}$
Cravat,	.75	=	.75		.75
	Ans.		\$15.250		\$15.25

2. Purchased 1 box of candles, for \$7.50; 1 box raisins, \$$3.37\frac{1}{2}$; 1 keg of buckwheat, \$$2.62\frac{1}{2}$; 1 barrel of flour, \$$9.87\frac{1}{2}$: what was the amount of the purchase ?

3. Bought 1 bag of coffee for \$$15.62\frac{1}{2}$; 5 sacks of salt, \$$4.37\frac{1}{2}$; 1 barrel of molasses, \$14.45, and 1 box of starch, \$3 $37\frac{1}{2}$: what was amount of bill ?

4. If you owe to A \$437.50; to B \$65; to C \$5.37½; to D 62½: what is the whole amount?

5. If your father's State tax is \$19.37½; his town tax \$25.12½; his poor tax \$6.62½, and his bridge tax \$5.87½: what is the sum of the whole?

6. Add \$125, \$65.37½, \$60.62½, and \$1235.87½ together.

Subtraction of American currency.

125. The *Subtraction* of American money is substantially that of simple numbers.

Example 1.—If my income is \$2500 a year, and my expenditures are \$2437.50: what is the surplus?

Explanation.

OPERATION.	EXPLANATION.
2500.00 2437.50 ——— *Ans.* \$62.50	Having for convenience put the two ciphers in the place of cents in the minuend, we say 0 from 0, nothing, which set as a cipher in the units-

place of cents, we then say 5 from ten (Art. 40, remark) 5; placing this in the tens-place of cents, we carry the 1 borrowed to 7, and then proceed with the subtraction as in the rule for simple numbers.

2. A man buys a horse for \$87.50, what change is he to receive from a hundred dollar bill handed the seller?

3. If you pay for a carriage \$450, and for a pair of horses \$337.75, how much more does the carriage cost than the horses?

4. What is the difference between \$775.37½, and \$562.12½?

5. What would \$595 deducted from \$1000.50 leave?

6. How much more is \$2000.60 than \$999.99?

7. Deduct \$735.39 from \$862.21, and state the remainder?

8. What is the difference between \$59.69, and \$96.95?

The multiplication of American currency.

126. The *Multiplication* of American money is similar in process to that of simple numbers.

Example 1.—What will 28 pieces of cotton bagging cost at \$15.50 a piece?

When cents are in the multiplicand.

OPERATION.	
15.50 28 ——— 12400 3100 ——— *Ans.* \$434.00	Remark.—When the multiplicand, as in this example, has cents, the two right hand figures in the result must be separated by the point for cents.

2. What is the cost of 40 barrels of flour at $6½ a barrel?

OPERATION.	
40	REMARK.—Here 40 × ½ = forty halves = 20 wholes; or it could be said $6½ = $6.50
6½	40
240	$260.00
20	
$260	

2 What is the cost of a firkin of butter, containing 84lbs., at 25½ per* lb?

3. What will 350 bushels rough rice cost at .87½ per bushel?

REMARK.—If this operation is performed by writing .875, named, when so written, decimals, for the multiplier, which is 87 cents 5 mills, three figures on the right hand side, in the result, are to be marked off, the first on the extreme right, by a comma, for mills; the next two by a point for cents. Our preference is for the other form, as being in common use, and practically best. When the multiplication is by .875, or decimals, equal to 87c. 5m.

4. I purchased a flock of sheep, numbering 225, at $2¼ each: what did the whole cost?

5. What will 66 bushels of oats cost at .33⅓ per bushel?

6. What must I pay for 52 barrels of potatoes, at $3¼ per barrel?

7. What will 380 acres of land cost, at $15¼ per acre?

8. How much has to be paid for 20 railroad shares, valued at $95.87½ each?

127. To find the cost of articles sold by the 100 or 1000, after multiplying the quantity by the price, we cut off two figures on the right hand of the product, if the price be by the 100; and three, if by the 1000; the remaining figures represent the answer, in the same denomination, as the price. To find the cost of articles sold per 100 or 1000.

9. What will 5750 bricks cost at $10 per thousand?

$$\begin{array}{r} 5750 \\ 10 \\ \hline \end{array}$$

Ans. $57.500
or
$57.50 c. 0 mills.

---

* *Per*, the Latin particle, signifying for.

10. Bought a raft of boards, containing 3345 feet, at $12 per thousand; what did the same amount to?

11. What is the value of 3475 feet of timber at $2 per hundred?

12. What must be paid for 450 feet of boards at $8 per thousand?

Articles sold by the ton. To find the worth of articles sold by the ton: having multiplied by the given numbers, we strike off three figures from the right of the product, and divide the remainder by 2 for the answer. This answer will be in the same denomination as the price of a ton. The reason of this division is, because the ton consists of 2000lbs., and the example proposes a number less than that.

Explanation of the 13th sum.

13. What cost 1637 weight of blades, at $10.50 the ton?

OPERATION.

```
    1637
     1050
    -----
    81850
   16370
   ------
 2)1718,850
   ------
Ans. $8.59
```

14. What will be the cost of 2676lbs. of plaster, at $2.65 per ton?

15. What will 950lbs. of hay cost, at $12.50 per ton?

16. What will be the freight of 5678lbs. of iron, at $9 per ton?

17 What will be the cost, by railroad, from Charleston to Memphis, on an invoice of merchandize, weighing 8560 tons, at $7 per ton?

Division of American currency. **128.** The *Division* of American money is to be performed, as in simple numbers.

How to divide dollars. **129.** When the sum to be divided consists of dollars, annex two ciphers at the right, in the place of cents, placing, always, the separation point between the dollars and cents. The answer will be in dollars and cents.

What is done when there is a remainder. **130.** Should there be a remainder, it is to be expressed, fractionally, as in the following example; or if it be desirable to pursue the inquiry further, by annexing a cipher to the dividend, the next division will give mills, and so on.

The former, or fractional way, to be preferred. REMARK.—The first way is the preferable one, for the reason already given, that in business transactions, we do not write beyond dollars and cents.

EXAMPLE 1. Divide \$9.67 by 5.

OPERATION.

5)9.67

1.93 and 2 over, which fractionally written is $\frac{2}{5}$, making the answer \$1.93$\frac{2}{5}$. But if performed so as to have mills in the result, it would be done thus: 5)9.67,0

1.93,4

which is to be read \$1.93 cents, 4 mills.

Note.—This subject will be treated of in decimal fractions.

2. Divide \$535 by 17

OPERATION.	EXPLANATION.
17)535.00(31.47$\frac{1}{17}$ 51 — 25 17 — 80 68 — 120 119 — 1	It will be noticed in this example, that as no cents were given in the sum, two ciphers have been put in the cents place; while in the quotient, two figures at the right hand have been marked off, showing the answer to be \$31.47 and the fractional expression, $\frac{1}{17}$. Had three ciphers been annexed to the dividend instead of two, the figure on the extreme right would have been mills.

3. Divide \$25.44 by 16.
4. Divide \$536 by 145.
5. Divide \$1000 into 250 equal parts.
6. Divide \$6532.56 into 105 equal parts.
7 If \$575 be divided equally among 8 persons, what will be the share of each?
8 Bought 56yds. of straw matting, for \$20: what was that per yard.?
9. Hired a carpenter for a month of 26 working days, at \$22: what was the expense of his services a day?
10. Sold 20 bags of Sea Island cotton for \$1975: what was the worth of a single bag?
11. At \$6 the barrel, how much flour can be had for \$2.58?
12. At 75 cents per pound, how much tea can be bought for \$9?
13. How many barrels of apples can be bought for \$45.50, at \$3.50 per barrel?

14. How long, with the wages of $1.12½ a day, will it take a laborer to earn $40.50?

Note.—In this example (Art. 107, Note Ex. 1), because of the fractional ½, double both divisor and dividend, and then proceed.

15. For $47.50, how many yards of broad cloth can be had at $2.37½ per yard?

16. At $7.50 a ton, how many tons of coal can be bought for $255?

17 If a bag of coffee, containing 160lbs., cost $24: what will be the price of a pound?

MISCELLANEOUS EXAMPLES.

131. EXAMPLE 1.—Which costs the more, 25 bushels of oats at 75 cents per bushel, or 124 yards of calico, at 7 cents per yard? What is the difference?

2. What is the difference in the cost between 5 tons of coal at $7.50 per ton, and 12yds. of cloth, at $3.50 per yard.

3. How many acres of land at $3 each, may be bought with the value of 46yds. of cassimere at $3 per yard?

4. How many acres of land at $3.50 per acre, may be purchased with the value of 120hhds. of molasses at 30 cents per gallon.

5. What is the cost of 2lb. 6oz. 5pwt. of silver at 27 cents per pwt.?

Note.—Reduce to pennyweights and then multiply by 27. This gives the answer in cents.

6. What is the value of 3 tons, 9cwt. 2qr. 18lbs. of sugar, at 13 cents per lb.?

7. What is the price of 2 bushels and 3 pecks of rice at 25 cents a peck?

8. What will 5lb. 7oz. salts come to at 9 cents per lb?

9. A merchant sold a remnant of cloth for $25.50; there were 6yds.: what was that a nail?

Note.—Reduce 6yds to nails, and divide into 2550 cents.

10. What will 150 acres, 3rd. 18 perches amount to, at $1.40 per perch?

11. Bought 3 tons, 9cwt. of iron for $450: what was that per cwt.?

12. If 1 ton of hay cost $12½, what will 5 cost?

13. If 1 lamb cost $2.25, what, at the same rate, will 15 cost?

14. If 85 bushels of corn cost $63.75, what is that per bushel?

15. If a butcher purchase 17 beeves for $180, 25 sheep for $67.50; what is the value of all? What of the beeves and sheep each per head?

16. A farmer buys 36 sheep, and pays for them with 5 cows, valued at $15 each, and a wagon worth $79: what do the sheep cost each?

17 An estate valued at $21,000, is to be divided among 4 children, when the widow has received her portion of one-third. What are the shares?

18. Bought a plantation for $4500; and paid for it with 10 shares in the South Carolina Railroad, $100 a share; 20 in the Savannah Railroad, at $125 a share: what amount will be called for to meet the balance?

HIGHER ARITHMETIC.

PART SECOND.

COMPOUND NUMBERS.

A compound number. **132.** A *compound number* consists of two or more denominations of like character, as expressed in the tables of currency, weights and measures.

What are compound numbers. **133.** Pounds, shillings and pence are of this class, as before stated; also, dollars, cents, dimes, bushels, quarts, pecks, etc.; but these or any other denominational values cannot be united to form a common sum: thus, it is impossible to say, £5 and $5— What are not. although both money representatives—are £10 or $10, and so of other like numerical classes.

How compound addition is performed. **134.** In adding compound numbers, place always the different values or measures in columns of a similar class, and in the order of the tables, commencing at the right hand with the lowest value named. When set in order, as in English money, pounds under pounds, shillings under shillings, pence under pence, farthings under farthings, add the right hand column, and divide it by the number of this denomination, to make one of the next higher: the quotient is to be carried to the next column, but the remainder placed beneath the added column.

COMPOUND ADDITION.

135. The proof is the same as in simple addition (Art. 39).

ENGLISH MONEY

OPERATION.

	£	s.	d.	qr.
		20	12	4
EXAMPLE 1.—	15	10	6	3
	14	15	10	2
	10	5	9	3
	25	19	8	3
	35	4	7	3
Ans.	101	16	7	2
		2	3	3

EXPLANATION. Explanation of sum.

The amount of the first column is 14qr. =3d. 2qr. The 2qr are placed in the column of farthings, and the 3d. (the small figure below) carried to the pence column, making 43d.=3s. 7d. The 7d. is now placed in the pence column, and the 3s. added to the shillings column, making 56s.=£2 16s. The 16s. is placed in the shillings column, and the £2 added to the £, making the answer as above.

* *Note.*—It is convenient to put the division numbers above the columns, and the carrying ones below in small figures, as in above example.

	£	s.	d.	qr.
		20	12	4
2.	30	15	9	2
	16	14	8	3
	17	12	6	1
	19	5	4	3
	3	3	10	2
Ans.	£87	12s.	3d.	3qr.
		2	3	2

	£	s.	d.	qr.
3.	12	5	4	2
	5	19	11	3
	16	15	4	2
	18	6	9	3
	7	12	8	1

	£	s.	d.	qr.
4.	250	17	9	2
	500	16	8	3
	1250	9	0	3
	25	7	5	2
	35	6	4	1

	£	s.	d.	qr.
5.	36	9	5	3
	19	5	10	2
	45	15	11	3
	120	18	9	2
	16	12	8	1

	£	s.	d.	qr
6.	55	17	10	2
	16	15	9	1
	14	13	7	2
	5	12	5	1
	29	16	3	2

	£	s.	d.	qr.
7	20	19	10	1
	35	17	9	2
	18	12	8	3
	9	14	5	2
	10	15	7	3

DRY MEASURE.

	bu.	pk.	qt.	pt.
8.	5	3	7	1
	6	2	6	1
	15	3	7	1
	25	2	5	0

	bu.	pk.	qt.	pt.
9.	69	3	6	1
	25	2	7	0
	12	3	6	1
	5	2	4	1

LIQUID MEASURE.

	tun.	hhd.	gal.	qt.	pt.
10.	150	1	45	2	1
	25	3	37	3	0
	55	1	29	1	1
	66	2	15	2	0
	75	1	12	1	1

	tun.	hhd.	gal.	qt.	pt.
11.	5	2	25	3	1
	6	3	15	2	1
	15	2	35	3	0
	9	1	60	2	1
	7	0	25	3	1

AVOIRDUPOIS WEIGHT.

	tons.	cwt.	qr	lb.	oz.	dr
12.	14	[illegible]	2	15	13	5
		15	3	24	14	3
		12	[illegible]	22	12	15
		9	2	[illegible]	10	9
		10	[illegible]	11	9	8

	tons.	cwt.	qr	lb.	oz.	dr.
13.	22	17	2	20	15	14
	15	15	3	9	10	13
	5	12	2	8	9	12
	4	9	1	15	7	8
	16	10	2	22	5	7

APOTHECARIES WEIGHT.

	lb.	℥.	ʒ.	℈.	gr.
14.	24	7	2	1	16
	17	11	7	2	19
	36	6	5	0	7
	15	9	7	1	13
	9	3	4	1	9

	lb.	℥.	ʒ.	℈.	gr.
15.	25	2	2	1	16
	15	11	4	2	15
	14	4	7	1	19
	3	2	5	2	11
	2	10	4	1	16

TROY WEIGHT.

	lb.	oz.	pwt.	gr.
16.	45	0	17	0
	9	3	14	0
	11	0	3	0
	8	9	15	20
		4	16	17
	10	2	3	15
		8	0	16

	lb.	oz.	pwt.	gr.
17.	35	10	12	14
	40	9	15	13
	12	3	14	20
	6	7	8	10
	2	3	14	15
	5	9	17	18
	4	10	18	20

ALE AND BEER MEASURE.

hhd.	gal.	qt.	pt.
48	31	3	1
49	50	1	1
50	47	2	1
45	15	3	1
14	12	1	1
18	13	1	1
19	3	2	1

19.

hhd.	gal.	qt	pt.
75	25	3	0
3	15	1	0
40	9	2	0
56	14	1	0
2	7	3	0
9	10	2	0
7	8	1	0

CLOTH MEASURE.

yd.	qr	na.
25	2	3
37	3	2
40	3	1
150	2	1
49	3	3
27	2	1

21.

E. Fl.	qr	na.
125	2	3
27	3	2
45	2	1
90	3	2
39	2	3
48	3	2

22.

E. E.	qr	na.
55	2	3
84	3	2
54	2	1
36	1	2
65	3	2
56	2	3

LONG MEASURE.

deg.	mi.	fur.	po.	ft.	in.
210	15	5	15	10	2
41	14	3	16	9	9
9	25	3	20	14	2
36	12	2	19	13	4
16	7	1	7	12	2
12	5	2	6	8	4

24.

deg.	mi.	po.	ft.
312	9	5	9
25	31	2	5
60	50	3	3
19	39	2	5
14	25	3	6
6	37	2	8

LAND OR SQUARE MEASURE.

sq. yd.	sq. ft.	sq. in.
97	4	104
22	3	27
105	8	2
37	7	127

26.

sq. mi.	a.	r.	rd.	sq. yd.
2	60	3	37	25
6	375	2	25	21
7	450	0	31	20
11	30	1	25	19

CIRCULAR AND ASTRONOMICAL MEASURE.

s.	°	′	″
5	17	36	29
7	25	41	21
8	15	16	09

28.

s.	°	′	″
6	29	27	49
8	18	29	16
7	09	04	58

29.

s.	°	
5	25	25
6	14	23
7	08	05

SURVEYORS' MEASURE.

	mi.	fur.	ch.	rd.		mi.	fur.	ch.	rd.
30.	25	8	3	10	31.	35	5	2	9
	24	5	6	09		25	6	4	8
	23	4	5	8		23	7	5	7
	23	3	4	7		20	8	6	6

SOLID OR CUBIC MEASURE.

	cu. yd.	cu. ft.	cu. in.		c.	c. ft.	c.	c. ft
32.	65	25	1129	33.	87	9	65	8
	37	26	132		26	7	35	6
	50	1	1064		16	6	25	5
	22	19	17		19	5	15	7

TIME MEASURE.

	mo.	wk.	da.		yr.	da.	h.	min.	sec.
34.	9	3	2	35.	89	59	20	13	12
	3	2	1		25	40	10	12	37
	7	1	2		5	90	19	19	25
	6	5	3		6	5	4	15	20
	4	3	2		5	6	14	5	6

COMPOUND SUBTRACTION.

How compound subtraction is to be performed.

136. To find the *difference* between compound numbers, place the less number under the greater of similar denominations, and beginning at the right hand, subtract as in simple numbers. Should the figure in the subtrahend, or lower line, exceed the one above, add to the number in the minuend as many as it takes of that denomination to make one of the next higher, and then take the subtrahend from the upper figure or minuend so increased. Set down the remainder, and carry one to the next denominator in the lower line, and so on.

ENGLISH MONEY.

EXAMPLE 1.—From 45 pounds, 10 shillings, 6 pence, 3 farthings, take 25 pounds, 9 shillings, 5 pence, 2 farthings.

	£	s.	d.	qr.
	45	10	6	3
	25	9	5	2
Ans.	£20	1s.	1d.	1qr

2. From 35 pounds, 13 shillings, 7 pence, 2 farthings, take 25 pounds, 17 shillings, 10 pence, 3 farthings.

OPERATION.

	£	s.	d.	qr.
		20	12	4
	35	13	7	2
	29	17	10	3
Ans.	£5	15s.	8d.	3qr.

EXPLANATION.

Explanation of work in example.

In this example, the upper figure being less than the lower, 4 farthings=1 penny, are borrowed and added to the 2=6, and the 3 of the subtrahend being subtracted, the remainder, 3, is set down in the farthings place, and 1d.=4 farthings that was borrowed, is carried to the next figure, 10+1=11. As that also exceeds the number of the minuend, we borrow and add 12 pence to the 7=19, and say 11 from 19 is 8; and setting it in the pence place, carry the 1s.=12 pence that was borrowed to the next figure, 17+1=18. As that exceeds the upper figure, we borrow and add 20 shillings to that, and say 18 from 33=15. That being set in the shillings place, and the borrowed 1=20 shillings carried to the pounds, we have, after the next subtraction, the answer.

In a similar way perform all examples, taking care to operate with the numbers expressed in the tables that apply to the proposed sum.

		£	s.	d	qr
3.	From	50	15	4	3
	Take	37	14	5	2

		£	s.	d.	qr
4.	From	37	18	9	0
	Take	18	19	5	2

		£	s.	d.	qr
5.	From	25	19	9	2
	Take	5	16	8	3

		£	s.	d.	qr
6.	From	15	6	9	2
	Take	14	7	6	3

DRY MEASURE.

		bu.	pk.	qt.
7.	From	38	6	3
	Take	25	4	1

		bu.	pk.	qt.
8.	From	12	0	0
	Take	9	7	2

LIQUID MEASURE.

		hhd.	gal.	qt.	pt.
9.	From	13	0	0	0
	Take	2	39	2	1

		hhd.	gal.	qt.	pt.
10.	From	165	15	2	1
	Take	59	36	3	1

AVOIRDUPOIS WEIGHT.

		ton.	cwt.	qr.	lb.	oz.	dr
11.	From	45	17	3	24	15	9
	Take	7	18	2	14	13	7

		ton.	cwt.	qr.	lb.	oz.
12.	From	200	5	2	20	13
	Take	33	12	1	23	15

APOTHECARIES' WEIGHT.

		lb.	℥	ʒ	℈	gr.
13.	From	3	1	5	0	14
	Take		5	2	7	17

		lb.	℥	ʒ	℈	gr.
14.	From	4	0	4	0	0
	Take		7	5	2	14

TIME MEASURE.

		yr.	da.	h.	min.	sec
15.	From	95	89	16	15	15
	Take	75	57	23	30	17

		yr.	da.	h.	min.	sec
16.	From	59	0	0	0	0
	Take	13	6	15	20	45

Note.—In subtracting dates, 30 days is counted a month.

To find the difference in dates.

17. What is the difference in time between March 15, 1863, and January 13, 1869?

	yr.	mo.	d.
From	1869	1	13
Take	1863	3	15
Rem.	5	9	28

Note.—In this sum, January is called the 1st month, and put down 1; March the 3d, and put down 3.

18. Calculate the time from July 6, 1857, to December 9, 1860.

19. Calculate the time from May 5, 1859, to August 4, 1869.

20. Calculate the time from March 15, 1862, to May 19, 1865.

21. How long is it from September 9, 1861, to November 4, 1870?

COMPOUND MULTIPLICATION.

137. In *Compound Multiplication*, the multiplier is, without exception, an abstract number, that is, a number separate from any particular object; as 4, 9, 15, 25, etc.; hence the product is simply the multiplicand repeated as many times as the multiplier expresses. The multiplier an abstract number.

138. To perform compound multiplication: having placed the multiplier under the lowest denomination of the sum, multiply this and divide the product by the number it takes of that denomination to make one of the higher; having set down the remainder, carry the quotient to the product of the next denomination, and so on. To perform compound multiplication.

ENGLISH MONEY.

OPERATION.

	£	s.	d.	qr.
		20	12	4
EXAMPLE 1.—Multiply	5	7	9	2
By				6
Product,	£32	6s	9d.	0qr.
		2	4	3

EXPLANATION. Explanation of sum.

Here we say, first, 2 ×6=12=3d. As there is no remainder we write 0 in the farthings place and carry the 3d. to the next number, after its multiplication; thus 9×6=54 and+3=57; this divided by 12 gives 4s. and 9d. over. We then put 9d. in the pence place, and carry the 4s. to the shillings after its multiplication; thus, 7×6=42 and+4=46; this divided by the 20 gives £2 6s. over. The 6s then is put in the shillings place, and after the £5 has been multiplied by 6=30 the 2 is added, which finishes the sum.

The proof. Multiplication is proved by division; thus, 6)£32 6s. 9d. 0qr., is

£5 7s. 9d 2qr, the operation will be seen in compound division (Art. 140).

LONG MEASURE.

		rd.	yd.	ft.	in	b.c.
		40	5½	3	12	3
2. Multiplicand,		15	4	2	6	1
Multiplier,						4
Product,	1 fur.	23	2	1	1	1
				1	6	0
	1	23	2	2	7	1

In this example, when we reach the denomination of yards, we say $4 \times 4 = 16$ and 3 added 19; by the multiplication of this by $2 = 38$, and by the multiplication of the numbers to divide it, 5½ by $2 = 11$, 11 into $38 = 3$ and 5 over; $5 \div 2$, to bring it back to the right denomination, gives 2½yd.: one-half yard=1ft. 6in. must be added to the feet and inches.

LIQUID MEASURE.

	gal.	qt.	pt.	gi.
3. Multiply	5	3	1	2
By				6

	gal.	qt.	pt.	gi.
4. Multiply	8	3	1	3
By				7

AVOIRDUPOIS WEIGHT.

	tons.	cwt.	qr.	lbs.	oz.	dr.
5. Multiply	14	17	3	20	10	15
By						4

TROY WEIGHT.

	lbs.	oz.	pwt.	gr.
6. Multiply	50	7	14	19
By				9

	lbs.	oz.	pwt.	gr.
7. Multiply	25	7	0	11
By				8

DRY MEASURE.

	bu.	pk.	qt.	pt.
8. Multiply	30	3	6	1
By				10

	bu.	pk.	qt.
9. Multiply	25	2	5
By			3

ALE AND BEER MEASURE.

		hhd.	gal.	qt.	pt.			hhd.	gal.	qt.	pt.
10.	Multiply	75	37	3	1	11.	Multiply	37	25	2	1
	By				25		By				15

CIRCULAR AND ASTRONOMICAL MEASURE.

		s.	°	′	″			s.	°	′	″
12.	Multiply	7	28	26	48	13.	Multiply	6	17	25	14
	By				20		By				6

TIME MEASURE.

		yr	da.	h.	min.	sec.
14.	Multiply	20	65	20	50	30
	By					12

COMPOUND DIVISION.

139. *Compound Division* is the process by which we find how often a given number is contained in a dividend of compound values of similar nature. Compound division defined.

140. To perform the work, divide the highest denomination by the given number and set down the quotient; if there is a remainder, reduce it to the next lower denomination; to the result add the given number of that denomination and divide as before; and so on until the whole has been divided. How it is performed.

EXAMPLE 1.—Divide £32 6s. 9d. 0qr., by 6 (Art. 138).

OPERATION.

	£	s.	d.	qr.
		20	12	4
6)	32	6	9	0
Ans.	£5	7s.	9d.	2qr.
				6
Proof,	£32	6s.	9d.	0qr.

EXPLANATION.

£32÷6 gives a quotient of £5, and a remainder of £2. These £2 reduced to shillings and added to 6s. make 46s., which divided by 6, gives a quotient of 7s., and a remainder of 4s. These 4s. reduced to pence and added to 9d., make 57d., which Explanation of sum in compound division.

divided by 6 give 9d. and a remainder of 3d. These 3d. reduced to farthings, and, as there are no farthings to be added, divided by 6, give 2qr., which finishes the work.

ENGLISH MONEY.

2. Divide £474 17s. 9d. 2qr., by 4.
3. Divide £90 15s. 7d. 3qr., by 9.

AVOIRDUPOIS WEIGHT.

4. Divide 46t. 17cwt. 3qr. 15lb. 9oz. 7dr., by 10.
5. Divide 60t. 15cwt. 2qr. 20lb. 6oz. 5dr., by 12.

Note.—When the divisor exceeds 12, it is necessary to put down the work, as in the following example:

DRY MEASURE.

6. Divide 216bush. 3pk. 5qt. 1pt., by 25.

OPERATION.

```
    bush. pk. qt. pt.
           4   8   2
25)216     3   5   1(8bush. 2pk. 5qt. 1 8/25pt.
   200
   ---
    16 bush.
     4
    --
    67 pk.
    50
    --
    17 pk.
     8
   ---
   141 qt.
   125
   ---
    16 qt.
     2
    --
    33 pt.
    25
    --
     8
```

EXPLANATION.

Finding that 25 is contained 8 times in 216, we put it in the quotient and multiply 25 by 8= 200: this we subtract from 216 and have a remainder of 16 bushels. These reduced to pecks make, with the three in the sum, 67 pecks. Dividing by 25, and putting the 2, which are the pecks, in the quotient, we multiply the divisor by the 2 and subtract. The 17 that remain are pecks, and being reduced to quarts, we divide as before, and so on.

APOTHECARIES' WEIGHT.

7. Divide 456lb. 6℥ 2ʒ 1℈ 18gr., by 49.
8. Divide 125lb. 5℥ 1ʒ 2℈ 15gr., by 36.

CLOTH MEASURE.

9. Divide 45yd. 3qr. 3na., by 15.
10. Divide 85yd. 2qr. 2na., by 9.

LIQUID MEASURE.

11. Divide 149hhd. 45gal. 3qt. 2pt., by 50.
12. Divide 70hhd. 25gal. 2qt. 2pt., by 8.

LONG MEASURE.

13. Divide 34deg. 24m. 7fur. 20po. 11ft. 9in., by 30.
14. Divide 420deg. 37m. 4fur. 30po. 5ft. 3in., by 65.

LAND OR SQUARE MEASURE.

15. Divide 97sq. yd. 4sq. ft. 104sq. in., by 33.
16. Divide 80 sq. yd. 3sq. ft. 97sq. in., by 26.

TROY WEIGHT.

17. Divide 6lb. 15oz. 20pwt., by 9.
18. Divide 12lb. 18oz. 15pwt., by 35.

ALE AND BEER MEASURE.

19. Divide 12hhd. 50gal. 3qt., by 6.
20. Divide 3hhd. 47gal. 2qt., by 22.

DRY MEASURE.

21. Divide 6bush. 3pk. 7qt. 1pt., by 7
22. Divide 12bush. 3pk. 6qt. 1pt., by 18.

CIRCULAR AND ASTRONOMICAL MEASURE.

23. Divide 10cir. 25s. 30′ 40″, by 9.
24. Divide 9cir. 20s. 18′ 30″, by 33.

SURVEYOR'S MEASURE.

25. Divide 12m. 7fur. 9ch. 3rd. 20li., by 6.
26. Divide 25m. 6fur. 8ch. 2rd. 15li., by 25.

SOLID OR CUBIC MEASURE.

27. Divide 4c. 14c. ft. 25t. 24c. yd. 1400cu. ft., by 9.
28. Divide 5c. 15c. ft. 39t. 25c. yd. 1650cu. ft., by 28.

TIME MEASURE.

29. Divide 25yr. 335da. 21h. 45min. 55sec., by 6.
30. Divide 50yr. 260da. 19h. 35min. 40sec., by 36.

PROPERTIES OF NUMBERS.

Prime and composite numbers.

141. Numbers are of two kinds, *prime and composite.*

A prime number.

142. A *prime* is always divisible by itself and 1, but can be divided by no other whole number: 1, 2, 3, 5, 9, 13, etc., are of this order.

A composite number.

143. A *composite* number is one that, while divisible by itself, can be divided by other numbers: such are 4, 6, 8, 15, etc.

Numbers concrete and abstract.

144. Numbers, whether prime or composite, are *concrete* and *abstract.*

A concrete number.

145. A *concrete* number is that which is applied to a particular object; as when is said 5 bushels, 12 books, 20 years.

An abstract number.

146. An *abstract* number stands disconnected with any object, and is simply 3, 4, 5, 20, etc.

A further distinction of composite numbers.

147. Composite numbers are *perfect or imperfect.*

A perfect number.

148. A *perfect number* (of which nine only are known) is equal to the sum of all its integral factors; thus, 28 is of this class, for the sum of $1+2+4+7+14=28$.

An imperfect number.

149. An *imperfect* number is not equal to the sum of its factors; thus, 8 is an imperfect number, for the sum of its factors, $1+2+4$, is not equal to 8.

What are factors.

150. The *factors* of a number are the makers or producers of it; thus 5 and 6 are the factors of 30; 3 and 8, or 3 and 4 and 2 are the factors of 24.

Prime factors.

151. The *prime factors* of a number are the lowest figures whose continued product make the number; thus, the prime factors of 24 are 2, 2, 2 and 3; the prime factors of 36 are 2, 2, 3 and 3.

What are not prime factors.

152. Prime numbers being divisible by themselves, are not called prime factors.

An integer defined.

153. An *integer* or whole number is a unit, or a collection of units.

When a work is called exact.

154. When both the divisor and quotient are integers, the work is called exact.

An aliquot part.

155. A number contained in another an exact number of times is known as an *aliquot* part; thus 10 cents, $12\frac{1}{2}$ cents, $33\frac{1}{3}$ cents, and 50 cents, are aliquot parts of a dollar.

A measure of any quantity.

156. A *measure* of any quantity is contained in that quantity a certain number of times without remainder; thus, 3 is a measure of 6, and 8 of 24.

157. The *common measure* of two or more numbers is any number that will divide each of them without remainder; thus, 4 is a common measure of 12, 16, 24. The common measure.

158. The *greatest common measure* of two or more numbers is the greatest number that will measure each of them; thus, 6 is the greatest common measure of 12, 18 and 30. A measure is sometimes called a *sub-multiple*. The greatest common measure. A sub-multiple.

159. The *multiple of any quantity* contains that quantity a certain number of times without remainder; thus, 20 is a multiple of 5, and 18 of 3. A multiple of a quantity.

160. A *common multiple* of two or more numbers is any number that may be divided by each of them without remainder; thus, 48 is a common multiple of 4, 6 and 8. A common multiple.

161. The *least common multiple* of two or more numbers is the least number that is exactly divisible by each of the given numbers; thus, 24 is the least common multiple of 4, 6 and 8. The least common multiple.

162. The *reciprocal* of a number is the quotient arising from the division of a unit by that number; thus, the reciprocals of 4, 9 and 12, are $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{12}$. The reciprocals of a number.

PRIME FACTORS.

163. To find the prime factors of a number, divide the number by the least prime number that gives an exact quotient; then divide that quotient by the least prime number as before, and so on. The several divisors and the last quotient will be the prime factors of the given number. To find the prime factors.

EXAMPLE 1.—What are the prime factors of 42?

OPERATION.

$$\begin{array}{r} 2)42 \\ \hline 3)21 \\ \hline 7 \times 3 \times 2 = 42. \end{array}$$

2. What are the prime factors of 30? *Ans.* 2, 3 and 5.
3. What are the prime factors of 56?
4. What are the prime factors of 63?
5. What are the prime factors of 76?
6. What are the prime factors of 180? *Ans.* 2, 2, 3, 3 and 5.
7. What are the prime factors of 7684?

GREATEST COMMON DIVISOR.

To find the greatest common divisor.

164. To find the greatest common divisor or measure of two or more numbers: resolve each number into its prime factors, and the product of the factors common to each result will be the answer.

EXAMPLE 1.—What is the greatest common divisor of 18, 30 and 48?

OPERATION.

$18=2\times3\times3$
$30=2\times3\times5$
$48=2\times3\times2\times2\times2$

Here, it is seen that 2 and 3 are factors common to all the numbers, and, also that they are the only common factors; hence, their product, $2\times3=6$, is the greatest common divisor.

2. What is the greatest common divisor of 60, 72, 48 and 84?

OPERATION.

$60=2\times2\times3\times5$
$72=2\times2\times2\times3\times3$
$48=2\times2\times2\times2\times3$
$84=2\times2\times3\times7$

Here 2 is a factor more than twice in some of the given numbers, but, as it is a factor only twice in others, we can only take it twice to find the result.

3 What is the greatest common divisor of 24, 48 and 96?

4. What is the greatest common divisor of 25, 75, 90, 85, 100 and 125? *Ans.* 5.

5. What is the greatest common divisor of 48, 72, 120, 144 and 168?

What is to be done when the rule is not a guide.

When the numbers are not readily the subject of the rule given, divide the greater number by the less, and the first divisor by the remainder, and so on, until there is no remainder; the last divisor will be the answer.

6. What is the greatest common divisor of 28 and 40? *Ans.* 4.

OPERATION.

```
28)40(1
   28
   --
   12)28(2
      24
      --
       4)12(3
```

7. What is the greatest common divisor of 16 and 28? *Ans.* 4.

8. What is the greatest common divisor of 8 and 15? *Ans.* 1.

9. What is the greatest common divisor of 216 and 408? *Ans.* 24.

10. What is the greatest common divisor of 315 and 810?

THE LEAST COMMON MULTIPLE.

165. To find the least common multiple or dividend of two or more numbers: arrange these in a horizontal line, and divide by any prime number that will go into two or more of them exactly, and place in a lower line the quotients and undivided numbers. Proceed in the same manner until there is no prime number greater than 1 that will divide without remainder any two of the numbers. The figures beneath the lower line and the divisors multiplied together will give the answer. To find the least common multiple.

EXAMPLE 1.—Find the least common multiple of 3, 4 and 8. *Ans.* 24.

OPERATION.

2)3......4......8

2)3......2......4

3......1......2

$3\times1\times2\times2\times2=24.$

2. What is the least common multiple of 6, 8, 12, 18 and 24?

3. What is the least common multiple of 33, 44 and 55?

4. What is the least common multiple of 9, 11, 17, 19 and 21? *Ans.* 223839.

5. What is the least common multiple of 4, 14, 28 and 98?

6. What is the least common multiple of 2, 7, 5, 6 and 8?

7. What is the least common multiple of 4, 12, 20 and 24?

8. What is the least common multiple of 2, 7, 14 and 49?

MISCELLANEOUS EXAMPLES.

166. Example 1.—From £2 10s. take 17s. 9d. 3qr.

2. From an ingot of silver, weighing 5lb. 3oz. 15dwt., six silver spoons, each in weight, 2oz. 10dwt. 12gr., were made: what remained?

3. From some merchandize, weighing, 18t. 19cwt. 3qr., were sold, 15t. 14cwt. 2qr. 14lb.: what was left?

4. A merchant had 455t. of sugar, but sold 225t. 16cwt. 3qr. 20lb.: what was over?

5. A merchant sold from a piece of broad cloth, containing 30yd. 3qr., 16yd. 2qr. 3na.: what quantity had he left?

6. An importer of wine sold from an invoice of 38 tun, 1hhd. 18gal. 2qt. 1pt., 17 tun, 3hhd. 42gal.: how much had he left?

7. From 50hhd. 34gal. 2qt. of ale, were sold 29hhd. 36gal. 3qt.: what remained?

8. A planter divided a plantation, containing 869 acres, 3rd., into 5 parts: what was in each?

9. What is the 10th part of 1yr. 3mo. 2w?

10. What is the 5th part of 20h. 49m. 50sec?

11. If a bale of English cloth, containing 356yd., cost £3425 2s. 4d.: what is the price per yd.?

12. Bought 40 loads of wood, each measuring 1c. 3c. ft. 7cu. ft., at \$4.50 per cord: what was the entire quantity and cost?

13. If a ship sail $2° 15' 29''$ a day: what will she have sailed in 25 days?

14. If five laborers dig a ditch, 6rd. 5ft. deep, in 3 days, what will they have done in 15 days?

15. What will 6lb. 7℥ 5ʒ 2℈ 10gr. come to, at 5 cents per grain?

16. What is the cost of 2t. 12cwt. 3qr. 20lb. of sugar, at 15 cents per lb.?

FRACTIONAL ARITHMETIC.

PART THIRD.

VULGAR FRACTIONS

167. *A fraction* is one of the equal parts of a unit, or a collection of units. A fraction defined.

168. The fraction, under present consideration, called vulgar, from being in most *common use*, is expressed by *two numbers*, vertically placed, with a separating line; thus, $\frac{1}{2}$ one-half; $\frac{2}{3}$ two-thirds; $\frac{3}{4}$ three-fourths. Why vulgar fractions, and how expressed.

169. The number that denotes into how many parts the unit is divided is called the *denominator*. Its place is always *below the line*. Thus, in the fraction $\frac{1}{2}$, 2 is the denominator, and it denotes that 1 has been separated into two parts. The denominator defined.

170. The figure *above the line*, as it names or numerates (that is numbers) how many parts of a unit, when divided, are used, is called the *numerator*; thus, in the expression $\frac{1}{2}$, one part of the two parts into which the unit was separated is denoted to be used. The numerator defined.

171. Taken together, the numerator and denominator are called the *terms of the fraction*. Terms of a fraction.

Note.—For convenience of expression, we always say numerator and denominator.

172. A fraction is simply a peculiar form of writing a divisor and dividend, the denominator being the divisor, and the numerator the dividend; thus $\frac{9}{3}$ is the same as $9 \div 3 = 3$; or $\frac{1}{2}$ is the same as $1 \div 2 = \frac{1}{2}$. Here the value of the fraction is seen to be the quotient of the numerator divided by the denominator. Fractions are unperformed division.

A proper fraction.

173. Fractions are variously known. *A proper fraction* has a less number for its numerator than for its denominator, as $\frac{2}{5}$, $\frac{3}{4}$.

An improper fraction.

174. *An improper fraction* has either equal numbers in its terms, thus, $\frac{3}{3}$; or, is greater in its denominator; thus, $\frac{5}{3}$, $\frac{15}{4}$.

A simple fraction.

175. *A simple fraction* is that which has whole numbers in both numerator and denominator; thus, $\frac{1}{2}$, $\frac{5}{2}$.

A compound fraction.

176. *A compound fraction* is a fraction of a fraction, or two fractions connected by the word of; thus, $\frac{2}{3}$ of $\frac{4}{3}$.

A mixed number.

177. *A mixed number* is a whole number and a fraction united in one term; thus, $5\frac{2}{3}$, $16\frac{3}{4}$.

A complex fraction.

178. *A complex fraction* is one whole numerator or denominator, or both, and has a fraction or mixed number in one or each of its terms; thus, $\frac{5\frac{1}{4}}{6}$ $\frac{\frac{2}{3}}{5}$ $\frac{4}{3\frac{1}{3}}$ $\frac{9\frac{2}{3}}{3\frac{3}{4}}$

179. REMARK.—As the addition and subtraction of fractions involves some preliminary work, we commence with examples in *multiplication.*

To multiply a fraction by a whole number.

EXAMPLE 1.—Multiply $\frac{2}{5}$ by 3.

OPERATION.

$\frac{2}{5}\times3=\frac{6}{5}$ *Ans.*

It will be seen that the work is done by the multiplication of the numerator by the whole number.

2. Multiply $\frac{1}{4}$ by 3.
3. Multiply $\frac{2}{5}$ by 2.
4. Multiply $\frac{3}{15}$ by 4.
5. Multiply $\frac{8}{12}$ by 9. *Ans.* $\frac{72}{12}=6$.
6. Multiply $\frac{3}{17}$ by 8.
7 Multiply $\frac{4}{25}$ by 7.
8. Multiply $\frac{5}{30}$ by 6.
9. Multiply $\frac{3}{8}$ by 5.
10. Multiply $\frac{3}{7}$ by 7

Sometimes, when the numbers are larger, the work is simplified by dividing the multiplier by the denominator, and multiplying by the numerator; but, on the whole, the former method is preferable.

11. Multiply $\frac{3}{9}$ by 25. *Ans.* $\frac{75}{9}$.

By the division of the denominator $\frac{1}{9}$ of $25=\frac{25}{9}\times3=\frac{75}{9}$ $=8\frac{3}{9}$.

12. Multiply $\frac{4}{7}$ by 35.
13. Multiply $\frac{21}{23}$ by 53.
14. Multiply $\frac{11}{36}$ by 65.

Note.—When the numerator exceeds the denominator, divide by the denominator.

180. Example 1.—Multiply $\frac{2}{3}$ by $\frac{4}{5}$. To multiply one fraction by another.

OPERATION.

$$\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}\ \textit{Ans.}$$

This is done by multiplying the numerators together for a new numerator, and the denominators together for a new denominator.

2. Multiply $\frac{4}{9}$ by $\frac{7}{15}$.
3. Multiply $\frac{16}{17}$ by $\frac{18}{20}$.
4. Multiply $\frac{30}{40}$ by $\frac{6}{7}$.
5. Multiply $\frac{88}{109}$ by $\frac{15}{35}$.
6. Multiply $\frac{63}{219}$ by $\frac{40}{50}$.
7. Multiply $\frac{30}{90}$ by $\frac{2}{6}$.
8. Multiply $\frac{50}{60}$ by $\frac{9}{18}$.
9. Multiply $\frac{32}{25}$ by $\frac{9}{39}$.
10. Multiply $\frac{70}{100}$ by $\frac{20}{30}$.

EXAMPLES IN DIVISION.

181. Example 1.—Divide $\frac{30}{24}$ by 5.

OPERATION.

$$30\div5=\frac{6}{24}.\ \textit{Ans.}$$

Here the numerator is divided by the whole number, and the same denominator is placed beneath. To divide a fraction by a whole number.

2. Divide $\frac{48}{57}$ by 6.
3. Divide $\frac{39}{80}$ by 13.
4. Divide $\frac{81}{560}$ by 9.
5. Divide $\frac{54}{336}$ by 6.
6. Divide $\frac{90}{100}$ by 9.
7. Divide $\frac{88}{350}$ by 11.
8. Divide $\frac{144}{250}$ by 12.
9. Divide $\frac{225}{275}$ by 15.
10. Divide $\frac{400}{450}$ by 20.

Remark.—In most instances, the result required is more easily obtained by multiplying the denominator by the whole number; but, usually, the division of the numerator is preferable. Multiplying, in some instances, the denominator.

182. Example 1.—Divide $\frac{4}{5}$ by $\frac{5}{7}$.

OPERATION.

$$\frac{4}{5}\times\frac{7}{5}=\frac{28}{25}=1\frac{3}{25}.\ \textit{Ans.}$$

The division of a fraction by a fraction.

In this case, we invert the divisor, and multiply the numerators and denominators together.

2 Divide $\frac{8}{18}$ by $\frac{14}{5}$

3. Divide $\frac{14}{17}$ by $\frac{18}{56}$.
4. Divide $\frac{19}{28}$ by $\frac{15}{38}$.
5. Divide $\frac{59}{66}$ by $\frac{7}{16}$.
6. Divide $\frac{15}{45}$ by $\frac{3}{14}$.

7. Divide $\frac{18}{36}$ by $\frac{9}{19}$.
8. Divide $\frac{30}{50}$ by $\frac{5}{3}$.
9. Divide $\frac{25}{37}$ by $\frac{9}{4}$.
10. Divide $\frac{19}{39}$ by $\frac{8}{7}$.

REDUCTION.

183. EXAMPLE 1.—Reduce $\frac{12}{15}$ to its lowest terms.

To reduce a fraction to its lowest terms.

OPERATION.

$\frac{12}{15} \div 3 = \frac{4}{5}$ *Ans.*

Here, the terms of the fraction are divided by their greatest common divisor, 3. Sometimes repeated divisions are employed before the final result is obtained.

2. Reduce $\frac{15}{18}$ to its lowest terms.
3. Reduce $\frac{27}{36}$ to its lowest terms.
4. Reduce $\frac{36}{45}$ to its lowest terms.
5. Reduce $\frac{40}{50}$ to its lowest terms.
6. Reduce $\frac{45}{72}$ to its lowest terms.
7. Reduce $\frac{60}{70}$ to its lowest terms.
8. Reduce $\frac{66}{77}$ to its lowest terms.
9. Reduce $\frac{70}{175}$ to its lowest terms.

In this example, it will be noticed that 5 is common to both terms, and that $\frac{70}{175} \div 5 = \frac{14}{35}$; it will be also noticed that 7 is common to both the terms, obtained by the division of 5, and that $\frac{14}{35} \div 7 = \frac{2}{5}$, the answer.

10. Reduce $\frac{288}{2592}$ to its lowest terms.
11. Reduce $\frac{351}{1755}$ to its lowest terms.

OPERATION.

$\frac{351}{1755} \div 351 = \frac{1}{5}$ *Ans.*

12. Reduce $\frac{172}{1118}$ to its lowest terms.

184. EXAMPLE 1.—Reduce $\frac{96}{6}$ and $\frac{54}{8}$ to their equivalent whole or mixed number.

To reduce an improper fraction to a whole or mixed number.

OPERATION.

$96 \div 6 = 16$; and $54 \div 8 = 6\frac{6}{8}$ *Ans.*

Here, the numerators, as being in excess of the denominators, are divided by them for the required result.

2. Reduce $\frac{560}{9}$ to a mixed number.
3. Reduce $\frac{156}{12}$ to its whole number.
4. Reduce $\frac{49}{6}$ to a mixed number.
5. Reduce $\frac{5678}{14}$ to a mixed number.
6. Reduce $\frac{96540}{1514}$ to a mixed number.
7. Reduce $\frac{376}{19}$ to a mixed number.
8. Reduce $\frac{540}{27}$ to a whole number.

185. EXAMPLE 1.—Reduce $4\frac{3}{4}$ to 4ths.

OPERATION.

$4 \times 4 = 16 + 3 = \frac{19}{4}$ *Ans.*

Here, the whole number is multiplied by the denominator of the fraction; the numerator added to the result, and a new fraction made, by placing the denominator given, under the sum obtained as above

To reduce a mixed number to an improper fraction.

2. Reduce $65\frac{7}{8}$ to 8ths.
3. Reduce $35\frac{5}{9}$ to 9ths.
4. Reduce $40\frac{6}{10}$ to 10ths.
5. Reduce $53\frac{5}{12}$ to 12ths.
6. Reduce $875\frac{3}{6}$ to an improper fraction.
7. Reduce $7543\frac{31}{35}$ to an improper fraction.
8. Reduce $5690\frac{4}{6}$ to an improper fraction
9. Reduce $3501\frac{37}{225}$ to an improper fraction.
10. Reduce $675\frac{1}{2}$ to an improper fraction.

186. EXAMPLE 1.—Reduce $\dfrac{5\frac{2}{3}}{6\frac{3}{4}}$ to a simple fraction.

OPERATION.

$$5\tfrac{2}{3}=\tfrac{17}{5}$$
$$6\tfrac{3}{4}=\tfrac{27}{4}=\tfrac{17}{5}\div\tfrac{27}{4};\ \tfrac{17}{5}\times\tfrac{4}{27}=\tfrac{68}{135}\ \textit{Ans.}$$

To reduce a complex fraction to a simple one.

We here reduce the numerator and denominator of the complex fraction, each to a simple one; and, inverting the terms of one of the fractions, multiply numerators and denominators.

2. Reduce $\dfrac{8\frac{4}{9}}{15\frac{3}{8}}$ to a simple fraction.
3. Reduce $\dfrac{3\frac{4}{13}}{5\frac{6}{7}}$ to a simple fraction.
4. Reduce $\dfrac{6\frac{3}{4}}{8\frac{2}{3}}$ to a simple fraction.
5. Reduce $\dfrac{\frac{6}{14}}{9\frac{3}{5}}$ to a simple fraction.
6. Reduce $\dfrac{5\frac{3}{8}}{\frac{3}{4}}$ to a simple fraction.
7. Reduce $\dfrac{4\frac{2}{3}}{\frac{3}{5}}$ to a simple fraction.
8. Reduce $\dfrac{\frac{9}{2}}{5}$ to a simple fraction.

OPERATION.

$$\frac{\frac{9}{2}}{5}=\frac{9}{2}\times\frac{5}{5}=\frac{\frac{45}{10}}{5}=\frac{45}{2}\ \textit{Ans.}$$

Note.—A whole number is divided by a fraction, by multiplying the whole number by the denominator, and then dividing the product by the numerator.

9. Reduce $\frac{8}{\frac{3}{4}}$ to a simple fraction.

10. Reduce $\frac{6}{\frac{2}{7}}$ to a simple fraction.

To reduce fractions to a common denominator.

187. EXAMPLE 1.—Reduce $\frac{3}{5}$, $\frac{5}{8}$ and $\frac{4}{5}$ to a common denominator.

OPERATION.

$3\times8\times5=120$, 1st numerator,
$5\times5\times5=125$, 2d numerator.
$4\times5\times8=160$, 3d numerator,
$5\times8\times5=200$, common denominator;
$\frac{120}{200}$, $\frac{125}{200}$, $\frac{160}{200}$. *Ans.*

Here, we multiply the numerator of each fraction by all the denominators, except its own, for the new numerators; and all the denominators together, for a common denominator.

2. Reduce $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$ to a common denominator.
3. Reduce $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{3}{8}$ to a common denominator.
4. Reduce $\frac{2}{5}$, $\frac{3}{6}$ and $\frac{4}{7}$ to a common denominator.
5. Reduce $\frac{7}{8}$, $\frac{4}{15}$, $\frac{9}{16}$ and $\frac{4}{6}$ to a common denominator.
6. Reduce $\frac{4}{5}$, $\frac{3}{11}$, $\frac{5}{7}$ and $\frac{2}{3}$ to a common denominator.
7. Reduce $\frac{3}{4}$, $\frac{2}{1}$, $\frac{5}{8}$ and $\frac{4}{7}$ to a common denominator.
8. Reduce $\frac{2}{10}$, $\frac{4}{7}$, $\frac{5}{9}$ and $\frac{2}{7}$ to a common denominator.
9. Reduce $7\frac{1}{2}$, $\frac{5}{4}$ and $\frac{2}{3}$ to a common denominator.

Note.—When there is necessity, as in this example, reduce to simple fractions.

10. Reduce $4\frac{1}{2}$, $5\frac{2}{7}$ and $1\frac{1}{2}$ to a common denominator.

To reduce fractions to the least common multiple.

188. EXAMPLE 1.—Reduce $\frac{3}{8}$, $\frac{5}{12}$, $\frac{7}{18}$ and $\frac{11}{24}$ to their least common denominator.

OPERATION.

2)	$\frac{3}{8}$	$\frac{5}{12}$	$\frac{7}{18}$	$\frac{11}{24}$
2)	4	6	9	12
2)	2	3	9	6
3)	1	3	9	3
		1	3	1

$\frac{72}{8}\times3=27$, 1st numerator.
$\frac{72}{12}\times5=30$, 2d numerator.
$\frac{72}{18}\times7=28$, 3d numerator.
$\frac{72}{24}\times11=33$, 4th numerator.
$2\times2\times2\times3\times1\times1=72$, least common multiple of denominators.

Hence, $\frac{3}{8}$, $\frac{5}{12}$, $\frac{7}{18}$ and $\frac{11}{24}=\frac{27}{72}$; $\frac{30}{72}$, $\frac{28}{72}$ and $\frac{33}{72}$. *Ans.* Here, as the fractions are in the lowest terms, we find the least common multiple of the denominators for a common denominator. Then, dividing this by each denominator, we multiply each quotient by its own numerator.

2. Reduce $\frac{5}{6}$, $\frac{7}{12}$, $\frac{3}{8}$ and $\frac{1}{4}$ to a common denominator.

3. Reduce $\frac{3}{12}$, $\frac{5}{14}$, $\frac{7}{25}$ and $\frac{2}{35}$.

4. Reduce $\frac{1}{8}$, $\frac{2}{11}$, $\frac{5}{66}$ and $\frac{6}{57}$.

5. Reduce $\frac{3}{7}$, $\frac{6}{20}$, $\frac{16}{30}$ and $\frac{8}{15}$.

6. Reduce $\frac{7}{15}$, $\frac{7}{18}$, $\frac{6}{25}$ and $\frac{3}{35}$.
Ans. $\frac{1470}{3150}$, $\frac{1225}{3150}$, $\frac{756}{3150}$ and $\frac{270}{3150}$.

7. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{9}$ and $\frac{7}{12}$.

8. Reduce $\frac{3}{11}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{8}$.

9. Reduce $\frac{2}{5}$, $\frac{3}{10}$, $\frac{7}{20}$ and $\frac{9}{30}$.

10. Reduce $3\frac{1}{8}$, $\frac{4}{5}$ and $\frac{5}{9}$.

189. EXAMPLE 1.—Reduce $\frac{5}{24}$ of a shilling to the fraction of a farthing.

To reduce a fraction of a higher to one of a lower denomination.

OPERATION.

$$\frac{5}{24}s.=\frac{5}{24}d.\times 12d.=\frac{60}{24}=\frac{5}{2}\times 4qr.=\frac{20}{2}qr.\ \ Ans.$$

We multiply the fraction to be reduced by such numbers as are necessary to the result.

2. Reduce $\frac{7}{600}$ of a pound Troy, to the fraction of a grain. *Ans.* $\frac{336}{5}$.

3. Reduce $\frac{7}{600}$ of a pound Apothecaries' weight, to the fraction of a grain. *Ans.* $\frac{336}{5}$.

4. Reduce $\frac{2}{265}$ of a day to the fraction of a second.

5. Reduce $\frac{2}{7}$ of a hhd. to the fraction of a quart.

6. Reduce $\frac{3}{19}$ of a bushel to the fraction of a gill.

7. Reduce $\frac{5}{9}$ of a yd. to the fraction of a nail.

8. Reduce $\frac{2}{7}$ of a cord to the fraction of a cu. ft.

9. Reduce $\frac{1}{23}$ of a gallon to the fraction of a pint.

10. Reduce $\frac{1}{6}$ of a mile to the fraction of a furlong.

190. EXAMPLE 1.—Reduce $\frac{32}{3}$qr. to the fraction of a shilling.

To reduce a fraction from a lower to one of a higher name.

OPERATION.

$$\frac{32}{3}qr.=\frac{32}{3}d.\div 4=\frac{8}{3}d.=\frac{8}{3}\div 12=\frac{8}{36}s.\ \ Ans.$$

The fraction is divided, it will be seen, by the numbers necessary to find the required denomination.

2. Reduce $\frac{225}{5}$gr. to the fraction of a pound Apothecaries' weight.

3. Reduce $\frac{336}{5}$gr. to the fraction of a pound Troy weight. *Ans.* $\frac{7}{600}$.

4. Reduce $\frac{580}{7}$sec. to the fraction of an hour.

5. Reduce $\frac{240}{3}$ of a gill to the fraction of a gallon.

6. Reduce $\frac{16}{9}$ of a cubic foot to a cord.

7. Reduce $\frac{32}{3}$pt. to the fraction of a bushel.

8. Reduce $\frac{5}{4}$in. to the fraction of a yard.

9. Reduce $\frac{8}{7}$d. to the fraction of a £.

10. Reduce $\frac{27}{3}$oz. to the fraction of a pound.

To reduce a fraction of a higher denomination to whole numbers of a lower denomination.

191. Example 1.—Reduce $\frac{9}{24}$ of a shilling to pence and farthings.

OPERATION.

$$\tfrac{9}{24}s.=\tfrac{9}{24}\times 12=\tfrac{108}{24}=\tfrac{9}{2}=4\tfrac{1}{2}d. \text{ and}$$
$$\tfrac{1}{2}=\tfrac{1}{2}qr.\times 4=\tfrac{4}{2}=2qr.;\ \text{hence, } \tfrac{9}{24}=4d.\ 2qr.\ \textit{Ans.}$$

We here reduce the given fraction to one of the next lower denomination, by multiplying with the number leading to such result. The improper fraction obtained, being reduced to a mixed number, its fractional part was reduced to the next lower denomination by multiplying with the number necessary to such result.

Note.—When the improper fraction reduced is a whole number, the result required is obtained.

2. Reduce $\frac{3}{16}$ of a mile to furlongs, chains, etc.
3. Reduce $\frac{5}{14}$ of a yard to quarters, nails, etc.
4. Reduce $\frac{7}{16}$ of a month to hours, minutes, etc.
5. Reduce $\frac{21}{38}$ of a bushel to pecks, quarts, etc.
6. Reduce $\frac{1}{9}$ of a circle to signs, degrees, etc.
7. Reduce $\frac{5}{24}$ of a ton to cubic feet and inches.
8. Reduce $\frac{3}{18}$ of a hhd. to quarts, pints, etc.
9. Reduce $\frac{5}{24}$ of a cord to cubic feet and inches.
10. Reduce $\frac{7}{19}$ of a circumference to miles, furlongs and rods.

To reduce whole numbers of lower to fractions of a higher denomination.

192. Example 1.—Reduce 8d. and 2qr. to the fraction of a shilling.

OPERATION.

$$8d.+2qr.=34qr. \text{ and } 1s. \text{ (to which they are to be reduced)} =48qr.;\ \text{hence, } \tfrac{34}{48} \text{ is found } =\tfrac{17}{24}s.\ \textit{Ans.}$$

Here, the given quantity is reduced to the lowest denomination possible, for a numerator; and a unit of the higher denomination is reduced to a like denomination with the numerator, for a denominator.

2. Reduce 9ʒ to the fraction of a pound.
3. Reduce 2pk. 1qt. $1\frac{7}{19}$pt. to the fraction of a bushel.
4. Reduce 2oz. $4\frac{1}{2}$dr. to the fraction of a pound.
5. Reduce 5 cord feet, 8cu. ft. and $1036\frac{4}{5}$in. to the fraction of a cord.
6. Reduce 8ft. 576in. to the fraction of a ton.
7. Reduce 2fur. 25rd. to the fraction of a mile.
8. Reduce 15° 40′ to the fraction of a sign.
9. Reduce 50gal. 3qt. 1pt. to the fraction of a hogshead.
10. Reduce 5ch. 3rd. 20li. to the fraction of a furlong.

ADDITION OF FRACTIONS

The addition of fractions.

193. EXAMPLE 1.—Add $\frac{3}{14}$ and $\frac{7}{14}$ together. $\frac{10}{14}$. *Ans.*

OPERATION.

$$\frac{7}{14}+\frac{3}{14}=\frac{10}{14}.$$

The denominators being of the same order, the numerators are added and written over the common denominator.

Note.—When the fractions are compound, complex, or mixed numbers, reduce to simple fractions.

2. Add together $\frac{3}{15}$, $\frac{4}{15}$, $\frac{5}{15}$ and $\frac{8}{15}$.
3. Add together $\frac{5}{20}$, $\frac{9}{20}$, $\frac{10}{20}$ and $\frac{16}{20}$.
4. Add together $\frac{8}{17}$, $\frac{9}{17}$ and $\frac{7}{17}$. *Ans.* $\frac{24}{17}=1\frac{7}{17}$
5. Add together $\frac{3}{36}$, $\frac{12}{36}$, $\frac{15}{36}$ and $\frac{24}{36}$.
6. Add together $\frac{5}{6}$ and $\frac{7}{8}$.

OPERATION.

$\frac{5}{6}=\frac{40}{48}$; $\frac{7}{8}=\frac{42}{48}\div 2$ for the least common denomination, $=\frac{20}{24}$ and $\frac{21}{24}$; $=\frac{41}{24}$. *Ans.*

7. Add together $\frac{5}{8}$, $\frac{7}{12}$ and $\frac{5}{15}$.
8. Add together $\frac{3}{8}$, $\frac{4}{6}$, $\frac{7}{10}$ and $\frac{5}{14}$.
9. Add together $3\frac{3}{4}$ and $5\frac{7}{8}$. *Ans.* $9\frac{20}{32}$.

Note.—In adding mixed numbers, it is often more convenient to add the whole numbers by themselves, and then the fractions.

10. Add together $6\frac{2}{3}$, $7\frac{1}{7}$ and $4\frac{2}{5}$.
11. Add together $4\frac{1}{4}$, $5\frac{2}{3}$ and $6\frac{2}{7}$.
12. Add together $2\frac{1}{3}$, $\frac{1}{4}$, $\frac{6}{7}$ and $\frac{7}{8}$.
13. Add together $\frac{3}{4}$ of $\frac{4}{5}$, $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{1}{2}$.
14. Add together $\dfrac{3\frac{1}{4}}{5\frac{2}{3}}$, $\frac{7}{8}$ and $\frac{9}{10}$ of $\frac{6}{7}$. *Ans.* $2\frac{1047}{4760}$.
15. Add together $\dfrac{3}{4\frac{1}{3}}$, $\frac{4}{5}$ of $\frac{2}{3}$ and $\frac{2}{5}$.
16. Add together $\frac{1}{5}$s. and $\frac{1}{3}$d.

OPERATION.

$\frac{1}{5}s.+\frac{1}{3}d.=\frac{12}{5}+\frac{1}{3}d.=\frac{36}{15}d.+\frac{5}{15}d.=\frac{41}{15}d.=2\frac{11}{15}d.$ *Ans.*

17. Add together $\frac{2}{5}$lb. $\frac{1}{7}$℥ $\frac{2}{7}$ʒ and $\frac{3}{8}$℈
18. Add together $\frac{1}{3}$gal. $\frac{1}{5}$qt. and $\frac{2}{7}$pt. *Ans.* $3\frac{37}{105}$pt.

SUBTRACTION OF FRACTIONS.

The subtraction of fractions.

194. EXAMPLE 1.—From $\frac{3}{4}$ take $\frac{1}{4}$.

OPERATION.

$$\tfrac{3}{4}-\tfrac{1}{4}=\tfrac{2}{4}. \ \textit{Ans.}$$

When they have a common denominator.

Here, the denominators being of the same order, the difference of the numerators is found, and placed over the common denominator.

2. From $\frac{18}{25}$ take $\frac{7}{25}$.
3. From $\frac{19}{30}$ take $\frac{16}{30}$.

Note.—When the denominators are not alike, reduce them to a common denominator, and then subtract as above.

4. From $\frac{2}{3}$ take $\frac{1}{2}$.

OPERATION.

$$\tfrac{2}{3}=\tfrac{4}{6} \text{ and } \tfrac{1}{2}=\tfrac{3}{6}=\tfrac{1}{6}. \ \textit{Ans.}$$

5. From $\frac{2}{3}$ take $\frac{2}{5}$.
6. From $\frac{3}{7}$ take $\frac{2}{9}$.
7. From $\frac{5}{8}$ take $\frac{1}{6}$.
8. From $\frac{19}{75}$ take $\frac{14}{85}$.
9. From $\frac{3}{4}$ of $\frac{4}{5}$ take $\frac{1}{2}$ of $\frac{2}{3}$. *Ans.* $\frac{4}{15}$.
10. From $9\frac{1}{3}$ take $1\frac{3}{4}$.

When there are compound or complex fractions, or mixed numbers.

Note.—Whenever there are compound, complex or mixed numbers, bring them to their simplest forms, and then subtract.

11. From $23\frac{5}{16}$ take $15\frac{4}{19}$.
12. From 120 take $40\frac{3}{5}$.

Note.—Change the whole number to 5ths.

13. From 265 take $52\frac{3}{7}$.

MISCELLANEOUS EXAMPLES

195. Example 1.—If 1lb. of beef is worth $8\frac{1}{2}$ cents, what is the value of $9\frac{1}{2}$lbs.?

2. Bought a firkin of butter containing $93\frac{1}{2}$lb., and was charged $27\frac{1}{2}$ per lb.: what did it amount to?

3. When barley is worth $\$1\frac{1}{4}$ a bushel, what will $4\frac{1}{2}$ bushels cost?

4. If 25bbl. of flour cost $\$125\frac{1}{2}$, what will $6\frac{1}{2}$bbl. cost?

5. When corn is selling for $\frac{3}{4}$ dollar a bushel, what must be paid for $255\frac{1}{2}$ bushels?

6. If you wish to divide \$130 among some laborers, so that 7 of them should have each $\frac{7}{3}$ as much as each of the other 3, what would you give?

7. If $2\frac{1}{2}$yd. of cloth will pay for $33\frac{1}{4}$lb of candles, what quantity of cloth will pay for $7\frac{1}{2}$ times $33\frac{1}{4}$lb. of candles?

8. If for $\frac{6}{11}$ of a bushel of potatoes you pay 42 cents, how many could be had for $\$8\frac{17}{200}$?

Ans. $10\frac{1}{2}$bush.

9. How many cubic feet are there in $\frac{9}{11}$ of a cord of wood, and what is it worth at $\$5\frac{1}{2}$ per cord?

Ans. $104\frac{8}{11}$ft.; $\$4\frac{1}{2}$

10. 32 is $\frac{8}{9}$ of how many times $\frac{1}{3}$ of 12?

Ans. 9 times.

11. Sold $73\frac{1}{3}$bush. corn for $\$64\frac{1}{6}$: what will be the amount, at the same rate of 64bush.? *Ans.* \$56.

12. 28 is $\frac{7}{8}$ of how many times 8?

13. How many cubic feet in a box that is $6\frac{3}{8}$ft. long, $5\frac{7}{9}$ft. wide, and $3\frac{1}{5}$ft. deep?

OPERATION.

$$6\tfrac{3}{8}\times5\tfrac{7}{9}\times3\tfrac{1}{5}=\tfrac{51}{8}\times\tfrac{52}{9}\times\tfrac{16}{5}=\tfrac{42432}{360}=\tfrac{1768}{15}=117\tfrac{13}{15}.\ \textit{Ans.}$$

14. In a box that is $8\frac{3}{8}$ft. long, $4\frac{3}{5}$ft. wide, and $3\frac{2}{7}$ft. deep, how many cubic feet are there?

15. Of the inhabitants of a town in Alabama, $\frac{3}{8}$ are planters, $\frac{2}{7}$ merchants, $\frac{1}{5}$ students and professional men, $\frac{1}{9}$ mechanics, and 142 others variously engaged: what is the number? *Ans.* 5040.

16. If the cargo of a ship be worth \$72,000, and if $\frac{3}{7}$ of $\frac{1}{2}$ of $\frac{7}{9}$ of the cargo be worth $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{10}{14}$ of the ship: what is the value of the ship?

17 In a school in Georgia, $\frac{1}{2}$ the scholars study arithmetic, $\frac{1}{4}$ algebra, $\frac{1}{5}$ geometry, and the others, in number 10, study engineering: how many scholars are there? *Ans.* 200.

18. The factors of a certain number are $32\frac{1}{4}$, $15\frac{1}{7}$ and $19\frac{4}{5}$: what is $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{6}$ of the number?

Ans. $3223\frac{11}{70}$.

19. How much cloth that is $\frac{3}{5}$ of a yd. wide, will it take to line a cloak, containing $8\frac{1}{4}$ yd. which is $\frac{11}{12}$ of a yd. wide? *Ans.* $12\frac{29}{48}$ yd.

20. What is $\frac{2}{3}$ of a barrel of flour worth at $\$6\frac{3}{8}$ per barrel?

21. A man can build $33\frac{1}{3}$ rods of wall in $24\frac{1}{2}$ days, by laboring $12\frac{1}{2}$ hours per day: in how many days of $9\frac{4}{5}$ hours, will he build $1\frac{1}{2}$ times as many rods?

22. A garden whose breadth is 10 rods, and whose length is $1\frac{3}{5}$ times its breadth, has a wall $3\frac{1}{2}$ feet thick around it: what was the cost of digging a trench $2\frac{3}{4}$ feet deep, in which to lay this wall, at $\frac{3}{4}$ cent per cubic foot? *Ans.* $\$62.94\frac{3}{4}$.

23. The distance from the earth to the sun is about 95,000,000 miles: in what time would a railway car run that distance, at the speed of $37\frac{1}{2}$ miles an hour, allowing $365\frac{1}{4}$ days in a year?

Ans. 288yr. 363d. 13h. 20min.

24. What is the value of $\frac{9}{13}$ of a day?

Ans. 16h. 36min. $55\frac{5}{13}$.

DECIMAL FRACTIONS.

The decimal fraction defined.

196. *A Decimal Fraction* is one that has its denominator written in tenths, or similar classes of numbers.

How the denominator is known.

197. This denominator is not usually expressed, but it is known from the *annexation* to its figure 1 *of as many ciphers* as the decimal demands.

How a decimal fraction is distinguished.

198. A decimal fraction is distinguished from a whole number by a dot, known as the *decimal point*, placed at the left of the decimal; the first figure at the right of the point being tenths, the second hundredths, the third thousandths, etc.; thus $.5=\frac{5}{10}$; $.55=\frac{55}{100}$; $.055=\frac{55}{1000}$.

NUMERATION TABLE OF DECIMALS.

How the decimal table differs from the one of whole numbers.

199. In the decimal table, which is read from left to right, it will be seen—as tenth denotes—that the numbers decrease in value. In the table of whole numbers it is remembered that the value increases from right to left. Similar names to designate values are used, always placing, or supposing to be placed, the integer 1 where the unit belongs, with its distinguishing or separating point.

Decimal numeration table.

Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten millionths.	
.5							is 5 tenths.
.6	7						67 hundredths.
.0	9	9					99 thousandths.
.4	5	6	7	7			4567 ten thousandths.
.0	2	3	4	5			2345 hundred thousandths.
.0	0	4	7	8	9		4789 millionths.
0	0	3	4	5	6	7	34567 ten millionths.

Why decimal fractions are written in this way.

200. Decimal Fractions are thus written to avoid the confusion that arises from the denominators, when expressed. These denominators are always understood; thus, .67 may be read $\frac{6}{10}$ and $\frac{7}{100}$, or $\frac{67}{100}$, which is the equivalent expression.

Value not altered by annexing ciphers.

201. The value of a decimal is not altered by the annexation of a cipher or ciphers; thus, $\frac{6}{10}=\frac{60}{100}=\frac{600}{1000}$; equivalently, these are .6=.60=.600.

When its value is decreased.

202. The value of a decimal is decreased to $\frac{1}{10}$ of its first value when a cipher is prefixed to it, since it removes the figure one place from the decimal point; thus, .6=$\frac{6}{10}$, but .06=$\frac{6}{100}$, which is but $\frac{1}{10}$ of $\frac{6}{10}$.

Whole numbers and decimals written together.

203. Whole numbers and decimals can be written together and easily read when the decimal point is placed between.

Table of whole numbers and decimals together.

204. TABLE.

Billions.	Hundreds of millions.	Tens of millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten millionths.	Hundred millionths.	Billionths.
9	8	7,	6	5	4,	3	2	1	.	0	2	3	4	5	6	7	8	9
Whole numbers.										Decimals.								

205. When a mixed number has the decimal point, it must be as a fraction of a unit of the order represented by the preceding decimal figure; thus, in the mixed number .2½, the ½ is half of a tenth; in .22½, it is ½ of a hundredth.

NOTATION OF DECIMAL FRACTIONS.

Notation of decimal fractions.

Write in figures:

EXAMPLE 1.—Thirty-five hundredths. *Ans.* .35.

2. Fifteen thousandth.
3. Fifty-five tenths of millionths.

Note.—It saves confusion in writing whole numbers and decimals together, to place the word decimal before the fraction: also, when a decimal only is expressed.

4. Five hundred and decimal two thousandths.
5. Four thousand and decimal six thousandths.
6. Six hundred and fifty and decimal three thousandths.
7. Eighty-five and decimal seventy-six thousandths.
8. One hundred and twenty and decimal fifty thousandths.
9. Two hundred and sixty and decimal fifty-six tenths.
10. Three hundred and decimal six hundredths.
11. Decimal three thousandths.
12. Decimal two hundredths.

206. Addition, subtraction, multiplication and division of decimals are performed precisely as whole numbers. How addition, etc., is performed.

EXAMPLE 1.

Add together	4.56	2.	9.56721
	36.902		3.42501
Sum,	41.462		12.99222
Proof,	41.462		12.99222
3.	365.78432	4.	534.21095
	276.23521		60.51954
	6543.12678		6.05678
	5995.67842		.00525
	2.16105		325.60789
	13182.98548		
	222 12211		

REMARK.—Remember always to place tenths under tenths, hundredths under hundredths, thousandths under thousandths, etc., and the point in the sum total directly under the numbers of the work given to be done.

5. Add 2.49, 515.2, 676.0003 and 3.5759.
6. Add 1765.5, 52.394, 6.0865 and .9538.
7. Add 5514, 39.7, 678.0212, .0967 and 31.5.
8. Add .799, .861, .4869, .395, .56789 and .3366.
9. Add 29.59, 5000.004, 230.03 and 56.547.
10. Add decimal three hundred and three thousandths, thirteen, decimal forty thousandths, two

hundred and decimal two thousandths, thirty-five millions and decimal four millionths, decimal thirteen thousandths. *Ans.* 35000213.358004.

SUBTRACTION.

OPERATION.

207. Example 1.

	1.	2.
From	9.6542	36.05946
Take	8.5431	28.75437
Rem.	1.1111	7.30509
Proof,	9.6542	

	3.	4.
From	54.367890	69.057546
Take	39.43207565	33.556327

Note.—In examples like the 3d, place ciphers over the figures in the subtrahend that have none, and then say 5 from 10, 5, and carrying 1 for what was borrowed, say 7 from 10, 3.

4. From 965.43445 take 45.395426.
5. From 874.32154 take 3.4506077
6. From .895456 take .000543670.
7. From 1.50050000 take .60051789.
8. From 23.0567 take 22.14675.
9. From 67469. take .56543.
10. From fifty-four millions take decimal fifty-four millionths.

Note.—Let the pupil be careful to place the separating points one under the other.

MULTIPLICATION.

208. EXAMPLE 1.—Multiply .36 by .36.

What numbers in result to be pointed off.

OPERATION.

```
 .36
 .36
 ---
 216
108
----
.1296
```

REMARK.—In the multiplication of decimals observe, as in the 1st and 2d examples, to point off as many figures for decimals in the product as in both multiplicand and multiplier are of this kind.

2.

```
Multiply 56.425
By         2.5
     ---------
      28212500
    112850
    ----------
    141.0625
```

Note.—The ciphers annexed are merely for convenience. The value of the sum would be the same if the multiplier had been placed at the extreme right. Such is the preferable form; thus,

```
  56.425
     2.5
  ------
  282125
 112850
 --------
141.0625
```

3.

```
Multiply     .7284
By          .00023
          --------
             21852
            14568
        ----------
        .000167532
```

4.

```
   .5682
     .36
 -------
   34092
  17046
 -------
 .204552
```

5. Multiply 567 by 3.24.
6. Multiply .435 by 345.
7. Multiply 5.95 by 63.32.
8. Multiply .2356 by .3453.
9. Multiply 65.36 by 234.5.
10. Multiply .0006 by .0005.

Note.—When there are not as many decimal figures

When a deficiency of decimal figures exists in product. in the product as there are decimal places in multiplier and multiplicand, place as many ciphers at the left as the deficiency requires.

11. Multiply .15
By .05

.0075

12. Multiply .6250 by .08.
13. Multiply .5945 by 009.

To multiply by 10, 100, etc. To multiply by 10, 100, etc., move the decimal point as many places toward the right as there are ciphers in the multiplier. This makes each figure in the multiplicand 10 times what it previously was. Hence, the result is ten times as great as the multiplicand.

14. Multiply .0467 by 100. *Ans.* 4.67
15. Multiply .00454 by 1000.
16. Multiply 8.764 by 300. *Ans.* 2629.2.
17 Multiply .0004 by 5600.
18. Multiply 3.253 by 2.900.

DIVISION.

209. Example 1.—Divide .845 by .13.

The number of decimal places to be in quotient.

OPERATION.

```
.13).845(6.5 Ans.
     78
     --
      65
      65
      --
```

Note.—There must be as many decimal places in the quotient as the decimal places in the dividend exceed those in the divisor. The excess in this example is 1.

2.

```
2.4).1248(.052.
     120
     ---
       48
       48
       --
```

The excess here is 3.

3. Divide 2.3462 by 2.11.
4. Divide 94.0056 by .08.
5. Divide .17638 by .369.
6. Divide 61064.14 by .4506.

210. When there are more decimal places in the divisor than in the dividend, annex ciphers to make the work practicable. *When an excess of ciphers in divisor.*

7. Divide 1941.855 by .7846. *Ans.* 2.475.

8. Divide 26943.16 by .3527

211. Should the number of figures in the quotient be less than the excess of decimal places in the dividend over those of the divisor, prefix to the quotient the number of ciphers which the deficiency requires. *When the quotient number is less.*

9. Divide .08756 by 1.1. *Ans.* .0796.

10. Divide .009792 by .12. *Ans.* .0816.

212. When it is desirable to extend the division beyond the given numbers, ciphers can be annexed. When it is not wished to go beyond a certain figure, or when it can be continued beyond the given number, we annex the sign +: thus, 4 divided by .7=.571+ *When ciphers are annexed.*

11. Divide .306656 by .5 to the extent of thousandths.

OPERATION.

5).306656

.611+

12. Divide .58765 by .6 to hundredths.

Decimal fractions are divided by 10, 100, 1000, by moving the point as many places toward the left as there are ciphers in the divisor. By this, each figure in the dividend becomes only $\frac{1}{10}$ as much as it previously was, and so the result is correspondent; thus, 567.5÷10=56.75. *To divide decimals by 10, 100, etc.*

13. Divide 7846.987 by 1000. *Ans.* 7.846987

14. Divide .9567 by 300.

15. Divide .7986 by 500.

15. Divide .54789 by 6000.

REDUCTION.

213. Example 1.—Reduce $\frac{3}{4}$ to a decimal fraction.

Remark.—The value of a fraction is the quotient arising from the division of its numerator by the denominator. This value remains unchanged, though a cipher or ciphers be annexed. Thus, to reduce $\frac{3}{4}$ we annex, say three ciphers, and divide by its denominator 4.

OPERATION.

4).3000

.750

Here, as many decimals are pointed off in the result as the decimal places in the dividend exceed those in the divisor.

2. Reduce $\frac{3}{16}$ to a decimal.
3. Reduce $\frac{2}{25}$ to a decimal.
4. Reduce $\frac{4}{40}$ to a decimal.
5. Reduce $\frac{5}{24}$ to a decimal.
6. Reduce $\frac{1}{2}$ to a decimal.
7. Reduce to a decimal of three figures $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{16}$.
8. Reduce to a decimal of five figures $\frac{6}{15}$, $\frac{7}{16}$, $\frac{8}{24}$.

A decimal changed to a vulgar fraction.

214. A decimal fraction is changed to a vulgar fraction by writing beneath it its proper denominator; thus, $.5=\frac{5}{10}$; $.05=\frac{5}{100}$; $.055=\frac{55}{1000}$.

9. Reduce .15.
10. Reduce .350.
11. Reduce 3.5.
12. Reduce 5.65.
13. Reduce 6d. and 3qr. to the decimal of a shilling.

3qr.$=\frac{3}{4}$d.$=.75$; thus, 6d. 3qr.$=6.75$. *Ans.*

14. Reduce 10s. 5d. 1qr. to the decimal of a pound.

OPERATION.

4)1.00qr.	
12)5.2500d.	1qr.=.25d.; 5.25d.=.4375s.; 10.4375s.=£521875.
20)10.43750s.	
£521875. *Ans.*	

Explanatory remark on operation.

REMARK.—Ciphers are annexed to the lowest denomination, and its division performed by the number of that denomination which it takes to make one of the next higher. The quotient is annexed as a decimal to the next higher number to be reduced, and so on to the result.

15. Reduce 6oz. 18dwt. 15gr. to the decimal of a pound Troy weight. *Ans.* .577604166lb.
16. Reduce 6℥ 2ʒ 2℈ 2gr. to the decimal of a pound.
17. Reduce 5yd. 2ft. 6in. to the decimal of a rod.

OPERATION.

12	6.0in.
3	2.500ft.
$5\frac{1}{2}$	5.8333+yd.
2	2
11	11.6666+half yd.
	1.0606+rod. *Ans.*

18. Reduce 43a. 3r. 25rd. 20yd. 6ft. 30in. to the decimal of a sq. mile.

19. Reduce 25ft. 1675in. to the decimal of a cord.

20. Reduce 3qr. 3na. to the decimal of a yard.

21. Reduce 2pk. 6qt. 1pt. to the decimal of a bushel.

22. Reduce £421875 to shillings, pence and farthings.

Ans. 8s. 5d. 1qr.

Explanation of the work

OPERATION.	EXPLANATION.
£421875 20s. ――― 8.437500 12d. ――― 5.2500 4qr. ――― 1.00	The first multiplication changes the decimal of the pound to the shillings and its decimal of a shilling; the second multiplication changes the decimal of the shillings to pence, and so on. As many places are pointed off in each product for decimals as there are decimals in the example. The answer

is the several denominations on the left of the decimal point.

Note.—The ciphers omitted in the multiplication are to be counted with the other figures, to determine the position of the point. They were omitted for convenience.

23. Reduce .9375 of a gallon to qt. pt. and gi.

Ans. 3qt. 1pt. 2gi.

24. Reduce .7694 of an acre to rd., etc.

25. Reduce .84 of a lunar month to wk., etc.

Ans. 3w. 2da. 12h. 28min. 48sec.

9

MISCELLANEOUS EXAMPLES.

215. EXAMPLE 1.—A grocer sold 16 bushels of potatoes at 62.5 per bushel; 3 bags of coffee at $15.625 per bag; and $14\frac{1}{2}$ barrels of flour at $6.50 per barrel: what was the amount?

2. A housekeeper purchased 56lbs. of sugar at 12.5 per lb.; 15lb. of currants at 18.75 per lb.; 5lb. of almonds at 16.33 per lb., and 6lb. of starch at 6 25 per lb.: what was amount of purchase?

3. Bought a keg of butter, containing 96lb., at 21.5 per lb., but sold one half of it for 25 cents per lb.: what was paid for the whole and received for the half?

4. A merchant sold 45.2yd. of cloth at $9.50 per yd.: what was his gain, the cost to him being 5.555 per yd.?

5. An adventure has gained for 6 persons $175.035: what is the share of each?

6. A merchant bought $14\frac{1}{2}$hhd. of wine at $75.333 per hhd., and sold them at public vendue, when they brought only $60.50 a hhd., exclusive of commissions and other expenses $35: what was his loss?

7. Bought 19 barrels of flour for $95.125: what was the cost of a single barrel?

8. Imported 65 bags of coffee at a cost of $996.555: what was the cost of a bag?

9. A trader bought 12 barrels of sugar at $18.75 per bbl.: what was the cost of each barrel?

10. A bag of cotton, weighing 350lb., was shipped to Liverpool and sold at 33.33 per lb., American money. Deducting various expenses, $5.125, what did it net the shipper?

11. A factor sold 26cwt. 3qr. of rice, in Liverpool, for £2 10s. per cwt.: what was the amount?

12. Bought 30bush. 3pk. of wheat at $1.375 per bushel: what was amount of purchase?

13. Bought 2 barrels of flour at $$6\frac{1}{2}$ per bbl; 4 bushels of corn at $62\frac{1}{2}$ cents per bushel; 6lb. of coffee at $16\frac{2}{3}$ cents per lb.; 10lb. of sugar at $6\frac{1}{4}$ cents per lb., and 5lb. of butter at $18\frac{3}{4}$ cents per lb.: what did they amount to?

Note.—Change fractions to decimals.

14. What would be the cost of building 37m. 6fur. 22rd. of railroad at $8355.62½ per mile?

15. A merchant bought 20hhd. of tobacco at $57¾ per hhd., and sold them for $1535½: what was his profit?

16. What is the value of .575cwt. of coal at £3 6s. 6d. 2qr. per ton?

17. What is the value of 10 bales of Sea Island cotton, the average of the bales being 320lb., at 43¾ cents per lb.

18. What must be paid for 52 weeks' board, at the rate each week of $1.37½?

19. At 25 cents a bushel, what number of bushels can be had for $200½?

20. A piece of land is 39.5 rods long and 25.56 rods wide: what will it cost to wall it at 62½ cents per rod?

CIRCULATING DECIMALS.

Circulating decimal defined.

216. *A Circulating Decimal* is simply a decimal fraction, repeating invariably the same figure once or many times.

When a single repetend: when a compound: when mixed: when a finite part.

217. When confined to a single figure it is called a single repetend, as .333, etc.; beyond this, a compound one, as 010101, etc.; but when combined with other decimals preceding its expression, a mixed repetend, as .8333. The figure or figures preceding the repetend is called the finite part of the expression, as 8.

How the repetend is had.

218. A repetend is obtained by reducing a vulgar fraction to a decimal when there is any prime factor, except 2 and 5, in the denominator, and not numerator, as $\frac{1}{3}$=.333, etc.; $\frac{4}{7}$=555, etc.

The way to avoid repetition.

219. To avoid repetition in a simple number, the same is written once, with a point above it; thus, $\frac{4}{7}=.\dot{5}$.; but in a compound, the first and last figures are thus designated, $\frac{36}{333}=\dot{1}0\dot{8}$.

What is a perfect repetend.

220. When the number of figures in the repetend is one less than the number of units in the denominator of the equivalent vulgar fraction, it is called a perfect repetend; thus, $\frac{1}{7}$ gives the perfect repetend, $.\dot{1}4285\dot{7}$.

To find the value of a mixed repetend.

221. To find the value of a mixed repetend, ascertain the value of the finite part and of the repetend separately, and add the results.

When one-ninth is reduced to a decimal.

222. Example 1.—What is the value of $.27\dot{5}$?

$$.27\dot{5}=27+\frac{5}{900}=\frac{243}{900}+\frac{5}{900}=\frac{248}{900}=\frac{62}{225}. \textit{ Ans.}$$

Note.—When a decimal begins to repeat at the third place, the two first figures will be so many hundredths, and the repeating figure so many ninths of another hundred.

2. What is the value of $.34\dot{5}$?
3. What is the value of $.67\dot{1}2\dot{3}$?
4. Change .4444, etc., to a vulgar fraction. *Ans.* $\frac{4}{9}$.
5. Change .9999, etc., to a vulgar fraction.

Note.—When $\frac{1}{9}$ is reduced to a decimal it produces a

quotient of .1111, etc., that is the repetend $\dot{1}$; $\frac{1}{9}$ is the value of the repetend $\dot{1}$; the value of .333, etc., or the repetend $\dot{3}$, must be three times as much, or $\frac{3}{9}$; 4, $\frac{4}{9}$; 5, $\frac{5}{9}$; and 9, $\frac{9}{9}=1$. When a decimal begins to repeat at the third place.

6. Change .5333, etc., to a vulgar fraction.

Here, the figure $5=\frac{5}{10}$ and the remaining part of the fraction is $\frac{3}{9}$ of $\frac{1}{10}$, that is $\frac{3}{90}=\frac{1}{30}$; add these, $\frac{5}{10}=\frac{15}{30}+\frac{1}{30}=\frac{16}{30}=\frac{8}{15}$. *Ans.* $\frac{8}{15}$ changed back will give .5333.

7. Change .3744, etc., to a vulgar fraction.

8. Change .46355, etc., to a vulgar fraction.

9. Change .363636, etc., to a vulgar fraction.

When $\frac{1}{99}$ is changed to a decimal it produces .010101, etc. The decimal .030303, etc., is three times as much, and is $\frac{3}{99}=\frac{1}{33}$. The decimal .363636, etc., is thirty-six times as much, and is $\frac{36}{99}=\frac{4}{11}$. When one ninety-ninth is changed to a decimal.

10. Change $.\dot{1}764\dot{7}$ to a vulgar fraction.

11. Change $.\dot{2}\dot{4}$ to a vulgar fraction.

12. Change $.\dot{4}\dot{2}$ to a vulgar fraction.

13. Change $.\dot{7}\dot{2}$ to a vulgar fraction. *Ans.* $\frac{72}{99}=\frac{8}{11}$.

14. Change $.\dot{0}0\dot{3}$ to a vulgar fraction. *Ans.* $\frac{3}{999}=\frac{1}{333}$.

CONTINUED FRACTIONS.

223. *A Continued Fraction* is one which has for its numerator a unit, and for its denominator a whole number plus a fraction, and so on; thus, A continued fraction defined.

$\frac{1}{2}+\frac{1}{3}+\frac{1}{1}+\frac{1}{1}+\frac{1}{3}+$etc., is a continued fraction.

224. The investigation of the nature and properties of this fractional formula belongs to the study of Algebra, and, hence, we give simply a passing reference to the subject. Its investigation properly belongs to Algebra.

EXAMPLE.—Reduce $\frac{17}{43}$ to a continued fraction.

OPERATION.

$\frac{17}{43}=\frac{1}{2}+\frac{9}{17}$ and $\frac{9}{17}=\frac{1}{1}+\frac{8}{9}$; hence, $\frac{17}{43}=\frac{1}{2}+\frac{1}{1}+\frac{8}{9}$.

Again, $\frac{8}{9}=\frac{1}{1}+\frac{1}{8}$; hence, $\frac{17}{43}=\frac{1}{2}+\frac{1}{1}+\frac{1}{1}+\frac{1}{8}$.

EXPLANATION.

We here divide both terms of $\frac{17}{43}$ by 17, the numerator of the fraction, and get $\frac{1}{2}+\frac{9}{17}$; then dividing both terms of $\frac{9}{17}$ by 9, the numerator, we get $\frac{1}{1}+\frac{8}{9}$; substituting then this value of $\frac{9}{17}$ in the expression $\frac{1}{2}+\frac{9}{17}$,

we have $\frac{17}{43}=\frac{1}{2}+\frac{1}{1}+\frac{8}{9}$, etc.

225. In any continued fraction, $\frac{1}{2}+\frac{1}{1}+\frac{1}{1}+\frac{1}{9}$, the several simple fractions are known as integral fractions, because the denominators are integers; thus, $\frac{1}{2}$ in the above is the first integral fraction, $\frac{1}{1}$ is the second, $\frac{1}{1}$ is the third, and $\frac{1}{9}$ is the fourth. Sometimes we call $\frac{1}{2}$ the first approximating or converging fraction; $\frac{1}{2}+\frac{1}{1}$ the second, and so on.

DUODECIMALS.

Duodecimals defined. **226.** *Duodecimals* are concrete* fractions, and are chiefly used in the measurement of surfaces and solids.

How changed. **227.** They are added, subtracted and divided, as other compound numbers; but, in certain cases, they are multiplied differently.

How they decrease. **228.** Duodecimals *decrease* uniformly from the highest to the lowest denomination, by the constant divisor, 12.

The measures. **229.** The measures used for their change are the *inch* or *prime*, the *second* and the *third*.

The measures divided, each into twelfths. **230.** A foot divided into 12 equal parts, is, in each division, called an inch or prime; an inch, also similarly divided, is called a second; and a second, a third; thus,

1 inch or prime, marked $1'=\frac{1}{12}$ of a foot.
1 second, " $1''=\frac{1}{12}$ of $\frac{1}{12}$ or $\frac{1}{144}$ of a foot.
1 third " $1'''=\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ or $\frac{1}{1728}$ of a foot, and so on, for minuter divisions, when required.

What are indices. **231.** The distinguishing points are known as *indices*.

ADDITION.

EXAMPLE 1.—Add 3ft. 6′ 3″ 2‴ and 2ft. 1′ 10″ 11‴ together.

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* Concrete is a term applied to a particular object.

OPERATION.

		12	12	12
		′	″	‴
	3.	6.	3.	2
	2.	1.	10.	11
Ans.	5ft.	8′	2″	1‴
		1	1	

EXPLANATION.

We first say 11 and 2 are 13; as this exceeds the common measure, 12, it is divided by 12‴, and its remainder, 1, is put down, and the quotient, 1 carried to 10=11. We next add 11 and 3, and dividing by 12″, put down the remainder 2, and carry 1. The other figures, being less than 12, are added like simple numbers.

2. Add 8ft. 9′ 7″ and 6ft. 7′ 3″ 4‴

Ans. 15ft. 4′ 10″ 4‴

3. Add 10ft. 8′ 6″ and 5ft. 5′ 5″
4. Add 9ft. 9′ 5″ and 7ft. 2′ 7″
5. Add 15ft. 6′ 7″ and 6ft. 8′ 6″
6. Add 20ft. 9′ 8″ 6‴ and 3ft. 6′ 4″ 5‴

SUBTRACTION.

232. EXAMPLE 1.—What is the difference between 9ft. 3′ 5″ 6‴ and 7ft. 3′ 6″ 7‴

OPERATION.

		12	12	12
From	9	3′	5″	6‴
Take	7	3	6	7
Rem.	1ft.	11′	10″	11‴
Proof,	9	3	5	6

EXPLANATION.

As the 6 of the minuend is less than the 7 to be subtracted from it, we add 12, and say 7 from 18=11. That put down and carrying 1, for the borrowed number, to the 6, we say 7 from 17, which is the 5 added to 12, and put down the remainder, 10, and so on.

2. What is the difference between 40ft. 6′ 6″ and 29ft. 7‴?

3. What is the difference between 12ft. 7′ 9″ 6‴ and 4ft. 9′ 7″ 9‴? *Ans.* 7ft. 10′ 1″ 9‴

4. What is the difference between 15ft. 8′ 7″ 6‴ and 12ft. 7′ 8″ 5‴?

5. What is the difference between 19ft. 9′ 8″ 7‴ and 14ft. 5′ 9″ 8‴?

6. What is the difference between 30ft. 8′ 9″ 10‴ and 29ft. 9′ 8″ 11‴?

MULTIPLICATION.

How multiplication is performed.

233. The multiplication of decimals consists in multiplying each term of the multiplicand by each term of the multiplier, commencing with the highest unit of the multiplier and the lowest of the multiplicand, and making the indices of each product equal to the sum of the indices of the factors.

Explanation of work of multiplication.

EXAMPLE 1.—A board is 5ft. 5′ 4″ in length and 3ft. 5′ 4″ in breadth: what are the contents?

Ans. 18ft. 9′ 5‴ 4⁗.

OPERATION.

	12	12		
5	5′	4″		
3	5′	4″		
16	4′	0″		
2	3′	2″	8‴	
	1′	9″	9‴	4⁗
18ft.	9′	0″	5‴	4⁗

EXPLANATION.

In this work, as the 4″ × 3ft.=12″=1′ (12÷12=1), we set down 0 under the seconds, and add the 1′ to the next product of 5′×3ft. =16′, which, reduced by dividing by 12′=1ft. 4′, we then write down the 4′ and carry the 1 to the next product=16ft.

In the same way, we multiply the multiplicand by the next figure, 5′, and this line being written down, we multiply by the 4 in like manner. The products are then added for the answer.

2. How many square feet in a board 17ft. 6in. long and 1ft. 7 inches wide? *Ans.* 27ft. 8′ 6″

3. What quantity of boards will it take for a floor 14ft. 8′ 3″ long and 13ft. 6′ 9″ wide?

Ans. 199ft. 2′ 4″ 8‴ 3⁗

4. How many feet in a plank 12ft. 4′ long, 2ft. 3′ wide, and 4′ thick? *Ans.* 111ft.

OPERATION.

12	4′		length.
2	3′		width.
24	8′		
3	1′	0″	
27	9′	0″	
	4′		thickness.
111ft.	0′	0″	

5. What are the contents of a block of marble that is 8ft. 9′ 3″ long, 3ft. 2′ 4″ wide, and 2ft. 5′ 7″ thick? *Ans.* 69ft. 0′ 10″ 4‴ 5⁗ 1′′′′′

6. What number of cubic feet in a granite block 3ft. 9in. wide, 2ft. 3in. thick, and 12ft. 6in. long? *Ans.* 105ft. 5′ 7″ 6‴

7. How many square yards in the walls of a room 14ft. 8in. long, 11ft. 6in. wide, and 7ft. 11in. high? *Ans.* 46yd. 3′ 8″

8. What will it cost to plaster a room 20ft. 6′ long, 15ft. wide, and 9ft. 6′ high, at 24 cents per square yard? *Ans.* 25.18\frac{2}{3}$.

9. How many square yards of oil cloth will it take to cover an entry that is 16ft. 6in. long and 8ft. 7in. wide?

10. If a load of wood be 8ft. long, 3ft. 9in. wide, and 6ft. 6in. high, how much does it contain? *Ans.* 1c. 4c. ft. 3cu. ft.

11. How many feet of boards will it take to make 12 boxes, whose interior dimensions are 4ft. 5′, 3ft. 6′, and 2ft. 7in., the boards being 1′ thick?

12. How many solid feet in a stick of timber 25ft. 6in. long, 2ft. 7in. broad, and 3ft. 3in. thick?

13. How many cords in a pile that is 25ft. 7′ long, 5ft. 4in. high, and 2ft. 9′ wide?

14. What will a marble slab cost that is 7ft. 4′ long and 1ft. 3in. wide, at $1 per foot? *Ans.* 9.16\frac{2}{3}$.

DIVISION.

234. Example 1.—In 165′ how many feet?

165÷12=13ft. 9in. *Ans.*

2. In 260″ how many feet and inches?

3. In 4367‴ how many feet? *Ans.* 2ft. 6′ 3″ 11‴

OPERATION.

$$\begin{array}{r} 12)4367''' \\ \hline 12)363.\ 11''' \\ \hline 12)30.\ \ 3'' \\ \hline 2.\ \ 6' \end{array}$$

4. In 5280‴ how many feet?

5. In 28800‴ how many feet?

6. In 56450‴ how many feet?

ANALYSIS.

Analysis defined.

235. *Analysis*, in arithmetic, is the separation of a sum into its component parts, with the relative bearings of the numbers in that question to each other.

It dispenses with formal rules.

236. It offers a simple and practical method to perform an operation without the formality of rules; and it is very useful in aiding solutions when rules, as in practical questions, occurring daily, are not remembered, and cannot be conveniently referred to.

237. In analysis we reason step by step from the inquiry to the result.

EXAMPLE 1.—If 6cwt. of hay cost $12, what will 9cwt. cost? *Ans.* $18.

Explanation of sum.

The analysis is that one hundred weight costs one sixth as much as six hundred weight. Since 6cwt. cost 12, 1 costs $\frac{1}{6}$ 12=2; 9 costs 9 times as much as 1cwt., and that is 9 times $\frac{1}{6}$ 12=18.

2. If 9 men can dig a trench in 25 days, how many will it take to do the work in 5 days?

3. A ship's company have provisions to last 12 men 8 months: how long would these last 18 men?

4. A hare has 39 rods the start of a hound, but the hound runs 27 rods while the hare runs 24: how many rods must the hound run to overtake the hare? *Ans.* 351.

5. The United States commander in Fort Sumter had 2lb. of bread per day for each soldier, for 10 days; but, by private dispatches, learning that his government would relieve him soon, he wishes to stave off surrender 15 days: to do that, what must be the daily allowance, say to 80 men?

6. 24 is $\frac{3}{8}$ of what number?

7. 76 is $\frac{9}{16}$ of what number?

8. If $\frac{18}{30}$ of a cask of wine cost $54, what will 4 casks cost?

9. A man sold a watch for $56, which was $\frac{8}{6}$ of its cost: what did he gain by the sale?

10. A farmer being asked the number of his sheep, said that if he had as many more, $\frac{1}{2}$ as many more, and $2\frac{1}{2}$, he would have a hundred: how many did he have?

11. A pole is $\frac{2}{5}$ in the mud, $\frac{3}{7}$ in the water, and 6 feet
ɔove the water: what is the length?

12. Three men hire a pasture for \$66; A puts in 2
ɔrses 3 weeks, B 6 horses $2\frac{1}{2}$ weeks, C 9 horses $1\frac{1}{3}$ weeks:
hat ought each to pay? *Ans.* A \$12; B \$30; C \$24.

ANALYSIS OF THE ABOVE.

The pasturage of 2 horses for 3 weeks is the same as Explanation to analysis.
ıat of 1 horse 2 times 3 weeks, or 6 weeks; that of 6
ɔrses $2\frac{1}{2}$ weeks, the same as that of 1 horse 6 times $2\frac{1}{2}$
eeks, or 15 weeks; and that of 9 horses $1\frac{1}{3}$ weeks, the
ıme as that of 1 horse 9 times $1\frac{1}{3}$, or 12 weeks. The ad-
tion of the weeks together, $6+15+12=33$ weeks; hence,
's share of payment is $\frac{6}{33}$ of \$66 ($66\div33=2\times6=\$12$);
s $\frac{15}{33}$ of $\$66=\$30$; and C's $\frac{12}{33}$ of $\$66=\$24$.

REMARK.—The pupil should begin an analysis from the
ɜrm which is of the same name or kind as the required
ıswer.

13. The inheritor of an estate spent $\frac{1}{3}$ of it in 9 months,
ıd $\frac{1}{5}$ of the remainder in 12 months more, when he had
ıly \$4000 left: what was the estate when he received it?

14. Divide \$176.40 among 3 persons, so that A shall
ave twice as much as B, and C three times as much as
: what is the amount when so divided?

15. Two men had the same income. The first saved $\frac{1}{6}$
f his each year; but the second, by spending \$200 a year
ıore than the first, was, at the end of 5 years, \$160 in
ebt; what was the income? *Ans.* \$1344.

16. If it take 44 yards of carpeting, $1\frac{1}{4}$ yd. wide, to
over a floor, how many yards of the kind, $\frac{7}{8}$ wide, will it
ıke? *Ans.* $62\frac{6}{7}$.

17. If an acre of land cost $\frac{1}{6}$ of $\frac{3}{7}$ of $\frac{1}{5}$ of \$50 what will
$\frac{1}{2}$ acres cost? *Ans.* \$10.

18. If $31\frac{1}{2}$ gallons of ale are worth $\$9\frac{3}{5}$, what, at that
ate, will $5\frac{1}{4}$ cost? *Ans.* \$1.60.

PROPORTIONAL ARITHMETIC.

PART FOURTH.

RATIO AND PROPORTION, OR SIMPLE RULE OF THREE.

Ratio defined.

238. *Ratio* is that relation of one quantity or number to another of the same kind, by which is found their equality or inequality.

Numbers relatively viewed.

239. When numbers are relatively viewed, the one which measures the other is considered as the standard, and the quotient resulting from the division by the standard is the ratio or relative value.

How ratio is indicated.

240. Ratio is indicated by a colon (:) in the first term; by a double colon (::) in the second; and by a colon in the third (:); thus,

4:8::6:12; that is, 4 is to 8, as 6 is to 12.

The terms. The antecedents. The consequents. First and second couplets.

241. The two quantities compared are the terms of the ratio; the first terms, or 4 and 6, in the above expression, being the antecedents, and the second, 6 and 12, the consequents. Here, the antecedents are the standard. The 4 and 8 are known as the first couplet, and 6 and 12 the second.

242. When two couplets have, as in the example, the same ratio, their terms are proportional; hence,

The work of a proportion.

243. *A proportion* compares the terms of two equal ratios.

A ratio of equality. Of greater inequality; of less inequality.

244. When the antecedent equals the consequent, the ratio being one, it is called a ratio of equality; thus, 4:4 =1.; when it is greater, or more than one, it is known as a ratio of greater inequality; thus, 6:2=3; when it is less than one, a ratio of less inequality; thus, $2:10=\frac{1}{5}$.

245. When there is but one antecedent and one conse- A simple ratio.
tent, the ratio is said to be simple; thus, 12:4=3.

246. When the corresponding terms of two or more A compound ratio.
nple ratios are multiplied together, the ratio arising from
is called compound; thus,

	4: 2= 2
4:2=2	6: 3= 2
6:2=3	12: 4= 3
———	———

24:4=6 and 288:24=12 are compound ratios; and, as
e illustration shows, are equal to the product of the sim-
: ratios of which they are composed.

247. The 1st and 4th terms of a proportion are called The extremes; the means
e extremes; the 2d and 3d the means; and the product
the extremes is equivalent to that of the means; thus,
the proportion,

3:9::12:36; 3×36=108; and 9×12=108.

248. The 4th term of a proportion is found by multi- To find the fourth term.
ying the second and third terms together, and dividing
the first; thus, 3:9::12: ?; 9×12=108÷3=36, the
m sought.

EXAMPLE 1.—What is the 4th term in the proportion,

3: 9:: 4 ?
5:15:: 3 ?
6: 8::12 ?
8: 4:: 4 ?

REMARK.—Any term of a proportion can be found To find other terms.
ien the three others are given; for the product of the
tremes, divided by either mean, gives the other; and
e product of the means, divided by either extreme, gives
e other.

In the solution of problems, two of the three given num- Two of the numbers to be of corresponding nature.
rs must be of the same kind; the third like the one re-
ired, thus,

2. If 4 men build 8 rods of wall in a day, how many
ll 6 build? *Ans.* 12.

4 men : 6 men :: 8 rods : 12 rods, the number sought.

3. If a staff 6ft. long cast a shadow 12 feet, what shadow
ll be cast at the same time by a steeple 60 feet high?

4. If a staff 6 feet long cast a shadow 9 feet what is the
ight of a tower whose shadow, at the same hour, extends
8 feet? and what is the ratio?

Ans. 132ft.; Ratio, 22.

5. If board for 52 weeks amounts to \$182, what is it for 39 weeks? *Ans.* \$136.50.

6. If I pay for 48 yards of cloth \$67.75, what, at the same rate, will 144 cost? *Ans.* \$201.75.

7 If 350 soldiers require 11,250 rations of bread for a month, how many will be necessary for a garrison of 600? *Ans.* 18,000.

8. If 12 men can build a house in 20 days, how many can do the work in 5 days? and what is the ratio? *Ans.* 48 men; Ratio, 4.

9. If 80 gallons, in an hour, run into a reservoir that will contain 1400, and 30 run out, in what time will it be filled? *Ans.* 28 hours.

Solutions by analysis preferable.

REMARK.—A useful way to solve questions like these is by analysis. As in business operations such method is usually adopted, we would advise it as practically the better plan.

10. 3 bricklayers build 6 rods of a foundation in a day: how many rods would 5 build?

If 3 men build 6, 1 will build $\frac{1}{3}$ of 6=2; if 1 build 2, 5 will build 5 times 2=10.

11. If 25 men perform a certain work in 35 days, how long will it take 9 to do the same?

12. If 27lb. of butter will buy 45lb. of sugar, how much can be had for 36lbs. of butter?

13. If an engineer's salary for three years amount to \$3600, what will it be in 9 years?

14. If $\frac{2}{5}$ of a yard of cloth cost $\frac{7}{9}$ of a dollar, what will $2\frac{1}{2}$ yards cost? *Ans.* \$4.86.

Here we say, if $\frac{2}{5}$ cost $\frac{7}{9}$, $\frac{1}{5}$ costs $\frac{1}{2}$ of $\frac{7}{9}=\frac{7}{18}\times 5$ for the whole $=\frac{35}{18}\times 2\frac{1}{2}$, that is $\frac{5}{2}$, $=\frac{175}{36}$, which, by the annexing of two ciphers for cents, =4.86.

Note.—Let it be remembered that a mixed number, as in this sum, is to be reduced to a fraction.

15. What is the price of $6\frac{4}{5}$yd. of cloth, if $\frac{1}{2}$yd. cost \$$2\frac{1}{2}$?

16. What is the cost of $3\frac{3}{4}$doz. of wine, if $\frac{1}{2}$doz. cost \$$7\frac{1}{2}$?

17 What is the value of $35\frac{3}{4}$ barrels of ale, if $4\frac{1}{2}$ barrels cost \$15?

18. A merchant owning $\frac{3}{5}$ of a ship, sells $\frac{2}{3}$ of his share for \$15,000: what is the value of the ship?

19. How many yards of silk, $\frac{3}{4}$yd. wide, will it take to line 15yd. of camlet $\frac{1}{2}$yd. wide?

20. If, when flour is worth $12 per barrel, a 5 cent loaf weigh 4oz., what ought it to weigh when flour is worth $8 per barrel?

21. If the earth revolve 366 times in 365 days, in what time does it revolve once? *Ans.* 23h. 56$\frac{4}{61}$m.

22. If 62a. 3r. 2rd. of land cost £615 9s. 3d., what will 15a. 2r. 3rd. cost?

23. If $\frac{5}{14}$ of a ship cost $1350, what is $\frac{2}{7}$ of her worth?

24. Borrowed $250 for 6 months; for how long must $350 be lent to repay the favor?

25. If 12$\frac{1}{2}$yd. of silk, $\frac{3}{4}$yd. wide, will make a dress, how many yards of cambric, that is 1$\frac{3}{8}$ wide, will line the same?

Ans. 6$\frac{9}{11}$.

COMPOUND PROPORTION, OR DOUBLE RULE OF THREE.

249. *Compound Proportion* is an equality which embraces in its consideration simple and compound ratios; thus, Compound proportion defined

$$\left.\begin{matrix} 3:12 \\ 16:\ 2 \end{matrix}\right\} ::18:9 \text{ is a compound proportion, and}$$

48:24::18:9 is the same in a simple form.

250. Questions can be reduced to a simple form, and some of complicated expression are easily solved, when the terms have been carefully arranged.

EXAMPLE 1.—If 3 men in 4 hours can thresh 15 bushels of rice, in how many hours can 2 thresh 5?

Ans. 2.

By Simple Proportion.

OPERATION.	EXPLANATION.
2:3::4:6, and	It will be seen that in
15:5::6:2, *Ans.*	the first proportion the

Explain the question

amount of labor is not made the subject of inquiry, but the time that 2 men will take to do the work of 3 men. Finding that to be 6 hours, the second inquiry is, in what time 5 bushels can be threshed, if 15 bushels are in 6 hours, and the time is found to be 2.

2. If $100 gain $6 in 12 months, what will be the gain of $3400 in 18 months? *Ans.* $306.

OPERATION.	EXPLANATION.
100:3400 :: 6:204 and 12 : 18::204:306	Here are stated three of the given things in a sim-

ple proportion; that $100 are to $3400 as $6, the interest, are to the answer: or, if $100 gain $6, what will $3400 gain? 100:3400::6:204. This being done, there remain the terms 12 months and 18 months to be disposed of, and forming a second proportion under the first, we say, 12 months are to 18 months::204 (the interest on 3400 for 6m.) :306 the answer.

Why called compound. The reason why questions conducted thus are called compound proportion is evident, for the answer sought is not only in proportion to the principal, but also in proportion to the time; and, therefore, in the compound proportion of the interest multiplied by the time.

How to solve questions. To solve questions in compound proportion, make for the 3d term that which is of the same kind or denomination with the answer. Then, take any two of the remaining terms that are alike, and arrange them as in simple proportion. In a similar way, arrange any other two terms of the same kind, and multiply the continued product of the 2d terms by the 3d term, and divide this result by the continued product of the 1st terms; the quotient will be the term required.

3. Ten men can build 25 rods of fence in 6 days: how many men will it take to build 30 rods in 3 days? *Ans.* 24.

OPERATION.	EXPLANATION.
25:30::10:12 and 3: 6::12:24 *Ans.*	With the statement of the first three terms, which

are in direct proportion, we have the two terms, 6 and 3, placed in what is called an inverse proportion to the answer, and in this way, because the more men are engaged, the less time is needed. These terms are therefore to be put inversely from the order necessary if the proportion had been direct. Instead of 6 to 3, say 3 to 6.

When the conditions are three or more. *Note.*—This same rule applies when there are three or more conditions to the question.

4. If a family of 6 persons spend $300 in 8 months, what amount is necessary to a family of 15 persons for 20 months? *Ans.* $1875.

5. If a family of 6 persons spend $600 in 8 months, how many dollars will be required for a family of 10 persons in 14 months? *Ans.* $1750.

6. A trader, with a capital of $300, gained $75 in 3 months: how much would he gain at the same rate with a capital of $1000, in 1 year (12 months)?

7 If 10 acres feed 15 head of cattle for 20 days, how many acres would feed 400 head for 90 days?

8. If 50 men can dig a ditch 100 yards long and 4 wide in 30 days, how many can dig one 400 yards long and 5 wide in 5 days?

9. A ship's company of 16 use 1hhd. of water in 1 month (30 days), how long, if the number be increased to 24, will 40hhd. last?

10. If 100 men can build a wall 300 feet long, 2 feet deep, and 6 feet high, in 10 days, in how many days can 50 men build 50 feet of wall, 3 feet deep and 4 feet high?

REMARK.—This somewhat complicated question may be stated thus, 300: 50::10:

2: 3

6: 4

50:100. The terms are in direct proportion to the answer, because more requires more; that is, the longer the wall, the longer the time for its completion; the deeper the wall, the longer time, and the higher, the longer time. The last proportion is inverse, because more men need less time, and it is the only one to be stated inversely.

11. If a twopenny loaf weighs 8oz. when wheat is 6s. 9d. per bushel, how much bread may be bought for 3s. 4d. when wheat is selling 13s. 6d. per bushel? *Ans.* 5lb.

12. If 40 men can dig a trench 40 yards long, 3 deep, and 8 yards wide, in 8 days, how many men would be employed to finish a trench 100 yards long, 12 wide, and 3 deep, in 24 hours (1 day)?

13. If 25 persons consume 300 bushels of corn in 1 year, how many bushels will 139 consume in 8 months; at the same rate? *Ans.* 1112.

14. If the cost per railroad of 12cwt. 3qr. for 400 miles, is $57.12, what will be the cost of 10 tons for 75 miles? *Ans.* $168.

COMMERCIAL ARITHMETIC.

PART FIFTH.

INTEREST

Interest defined.

251. *Interest* is a premium for the use of borrowed money.

Names connected with it.

252. The money on which interest accumulates is called the *Principal;* and the principal, with its interest, is called the *Amount.*

Premium different in different States.

253. The premium is a legalized value, but is not uniform in the Confederacy. In South Carolina, the rate or annual per centage is 7 per cent., that is, 7 cents on the 100 cents, or 7 dollars on the 100 dollars, for a year. In Georgia, it is 8 per cent.; in Texas, 12 per cent.; in some of the States of the old Union, 6 per cent.; in France and England, 5 per cent.

The terms per cent., and per annum.

254. The term per cent. (per centum) is Latin, and signifies for the hundred; and per an. (per annum) for the year.

Note.—A sum at simple interest becomes double in 16 years, 8 months.

How long it takes to double an amount.

255. Example 1.—The interest on $1, for a year, is 7 cents in South Carolina, what is the amount of interest due on $100 for the same period? *Ans.* $7.

OPERATION.		REMARK
Multiply	100	Remark.—The $100×7 =700; and as the multiplication of dollars by cents (Art. 121, Remark) gives cents, the answer is 700 cts.
By	7	
	700	

In multiplying dollars by dollars, let the pupil remember that the result will be dollars; but when dollars are multiplied by cents, as in the above example, the answer is in cents. Two right hand figures pointed off for cents, show the answer to be $7.

When dollars are multiplied by dollars, and dollars by cents

2. What is the interest of $50, for 1 year, at 7 per cent.? *Ans.* $3.50

OPERATION.

$$\begin{array}{r} 50 \\ 7 \\ \hline 350 \text{ cents.} \end{array}$$

3. What is the interest of $90, for 1 year, at 7 per cent.? *Ans.* $6.30.

4. What is the interest of $99, for 1 year, at 7 per cent.?

5. What is the interest of $250, for 1yr. 6m., at 7 per cent.?

Note.—The interest for 6m. (½yr.) is to be added to that of the year.

6. What is the interest of $300, for 1yr. 4m., at 7 per cent.? (4m.=⅓ of a year.) *Ans.* $28.

When the number of months is an equal part of 12, as in the last example (12÷3=4), it facilitates calculation to take an equal part of the year's interest, according to the question, and to add it to the amount for the year.

How in some cases calculation is facilitated.

Note.—The same applies to parts of a month and to days.

7. What is the interest of $700, for 2 years and 2 months, at 7 per cent.? (The 2m.=⅙ 12.)

8. What is the interest of $900, for 1 year, 3 months and 15 days, at 7 per cent.? (The 15d.=½ month.) *Ans.* $80.37½.

9. What is the interest of $1000, for 2 years, 1 month and 20 days, at 7 per cent.? (The 20 days is ⅔m.)

10. What is the interest of $75, for 1yr. 5m. and 18d., at 7 per cent.? (5m.=$\frac{5}{12}$ of 12; 18d.=$\frac{18}{30}$ of 30.)

11. What is the interest of $156, for 2 years, 8m. and 10d., at 7 per cent.? (8m.=⅔ 12; 10d. ⅓ 30, or ⅓m.)

12. What is the interest of $17.49, for 1yr., 7m. and 8d., at 7 per cent.? *Ans.* $1.685.

When there are cents in a question. *Note.*—Here, dollars multiplied by cents give cents, but cents multiplied by cents give or reduce to mills (Art. 121, Remark). Thus, the answer is \$1. 68c. 5m.=½ cent.)

13. What is the interest of \$459, for 5yr., at 8 per cent.? *Ans.* 183.60.

14. What is the interest of \$600.50, for 3yr., at 8 per cent.? *Ans.* \$144.13, 4 mills.

15. What is the interest of \$62, for 3yr., at 8 per cent.?

16. What is the interest on a Confederate State bond, for \$1000, for 2yr. 6m., at 8 per cent.?

17. What is the interest of \$156, for 1yr. 9m. and 15d., at 12 per cent.? (9m.=¾ 12m.)

18. What is the interest of \$256, for 6m. 20d., at 8 per cent.?

19. What is the interest of \$444, for 1yr. 10m., at 6 per cent.? (10m.=$\frac{10}{12}$ of 12.)

20. What is the interest of \$85.30, for 1 yr. 16d., at 6 per cent.?

21. What is the interest of \$550, for 90 days, at 7 per cent.? (90d.=3m.)

22. What is the interest of \$150, for 60 days, at 7 per cent.? (60d.=2m.)

23. What is the interest of \$125, for 30 days, at 7 per cent.? (30d.=1m.)

24. What is the interest of \$38.55, for 2yr., at 7 per cent.?

What to keep in mind in pointing off numbers. REMARK.—Let the pupil remember that when cents are in the question, 4 figures, counting left to right, are to be pointed off; and that the two, on the right of the point, are cents, and the others tenths; thus, as in the 14th example, the numbers in the result are 1441344, which, pointed off as directed, read \$144.13 cents, $\frac{44}{10}$, or 4 mills, etc.

25. What is the interest of \$95.50, for 1yr. 6m., at 7 per cent.?

26. What is the interest of \$350, for 3yr. 8m. 18d., at 6 per cent.? *Ans.* \$79.383.

REMARK.—In mercantile transactions it is customary to take for the answer only two figures, cents, at the right hand of the separating point. This answer would usually be read \$79.38.

27. What is the interest of $326, for 3yr. 2m., at 7 per cent.?

28. What is the interest on $56.52, from March 19, 1859, to January 25, 1862, at 7 per cent.?

29. What is the interest on $598, from July 15, 1860, to Oct. 20, 1864, at 7 per cent.?

30. What is the interest on $135, from Dec. 10, 1861, to May 17, 1863, at 7 per cent.?

31. What is the interest on $65.50, from Jan. 1, 1855, to Feb. 1, 1863, at 7 per cent.?

32. What is the interest on $100, from June 19, 1860, to June 10, 1865, at 7 per cent.?

33. What is the interest on £27 15s. 9d., for 1 year, at 5 per cent.? *Ans.* £1 7s. 9d. 1qr.

OPERATION.

```
£27 15s. 9d.=27.7875
                   5
             --------
             1.389375=£1.389375
                             20
                       --------
                       s. 7.7875
                             12
                       --------
                       d. 9.4500
                              4
                       --------
                       qr. 1.80
```

EXPLANATION.

As the principal is in pounds, shillings, pence, we reduce the lower denominations to the decimal of a pound (Art. 214, 14 Ex.), and multiply that, with the pounds prefixed, by the per centage. In this example, the left hand figure is the interest of the pounds denomination. The decimal interest is then reduced back to shillings, pence and farthings. It will be noticed that only 3 decimal places in the multiplicand are used.

34. What is the interest of £16 9s. 5d. 2qr., for 1yr. 6m. 15d., at 5 per cent.?

35. What is the interest of £75 12s. 9d. 3qr., for 2yr. 9m. 20d., at 5 per cent.?

36. What is the interest of £100 3s. 3d., from March 9, 1862, to June 24, 1864, at 5 per cent.?

37. What is the interest of £90 6s. 3d., from Dec. 1, 1862, to Oct. 11, 1863, at 5 per cent.?

38. What amount of principal and interest, at 7 per cent., is due Jan. 1, 1861, on a note of $400, dated Jan. 1, 1859; there having been paid, July 1, 1859, $100; Jan. 1, 1860, $150; and July 1, 1860, $50?

When payments have been made on a note.

REMARK.—In like examples cast the interest from the date of the note to the specified time, and add to the principal; then, on the several payments, from their dates to

the specified time, and add to their collected amount this credit interest; subtract the same from sum of principal and interest, for answer.

39. What was due on a note of \$448.50, at 7 per cent. interest, dated June 15, 1854, when finally settled July 3, 1856, which had these endorsements: Dec. 6, 1854, \$75; April 19, 1855, \$125; Dec. 15, 1855, \$10; Jan. 25, 1856, \$100? *Ans.* \$183.60.

40. What was due, March 26, 1855, on a note for \$1000, at 6 per cent. interest, on which had been paid, Sept. 6, 1850, \$50; July 14, 1851, \$150; Aug. 9, 1852, \$25; May 14, 1853, \$28; Oct. 15, 1853, \$125; Nov. 11, 1853, \$75; and Nov. 13, 1854, \$500? *Ans.* \$282.58.

To find the principal when the interest, time and rate are known.

If it be desirable to ascertain the principal, when the interest, time and rate alone are known, cast the interest on one dollar for the given time, and divide the interest by it.

41. The interest of a certain sum, for 2 years, at 7 per cent., is \$70: what is the principal? *Ans.* \$500.

OPERATION.

Int. on $\$1=7\times2$(2 years)$=14$; and $70.00\div14=500$.

42. The interest of a certain sum, for 1yr., at 7 per cent., is \$63: what is the principal? *Ans.* \$900.

43. The interest of a certain sum is \$350 a year, at 7 per cent.: what is the principal?

44. The interest of a certain sum, for 1yr., at 7 per cent., is \$1500: what is the principal?

To find the rate when interest, time and principal are known.

When the interest, time and principal are known, we ascertain the rate by casting the interest on the principal for the given time, at 1 per cent., and then dividing the known interest by it. The quotient is the rate.

45. The interest of \$500, for 1yr., is \$35: what is the rate? *Ans.* 7 per cent.

OPERATION.

500)35.00(7
 35.00

Here, to accomplish the division of 35 by 500, we annex (Art. 129) two ciphers to the \$35

46. The interest of \$1200, for 2 years, is \$144: what is the rate? *Ans.* 6 per cent.

47 The interest of \$1500, for 1 year, is \$120: what is the rate?

48. The interest of \$2000, for 1yr., is \$100: what is the rate?

To find the time when the principal, interest and rate are known.

When interest, principal and rate are known, we ascertain the time by casting the interest on the given principal, at the known rate, for 1 year, and dividing the

interest by it. The quotient will be the time in years and decimals of a year.

49 What is the time when the principal is \$900, the interest is \$63, and the rate is 7 per cent.?

Ans. 1 year.

OPERATION.

$$\begin{array}{l} \text{Int. } 900 = 63, \text{ and } 63 \div 63 = 1. \\ \quad\;\; 7 \\ \hline 63.00 \end{array}$$

50. What is the time when the principal is \$750, the interest \$90, and the rate 6 per cent.? *Ans.* 2 years.

51. For what time must \$200 be at interest, at 6 per cent., to gain \$36?

COMPOUND INTEREST.

256. *Compound Interest* is interest on principal and interest united and making a new principal. The latter being, as an unliquidated debt, added to the former. Compound interest defined.

257. No law recognizes such computation, but it is equitable and becomes legal when payment of due interest is not made on demand. In such case the unsettled interest becomes a principal, and interest, as on any debt similarly placed, can be charged. Its computation equitable.

Note.—An amount at compound interest doubles itself in 11 years, 8 months and 22 days. In what time a sum at compound interest is doubled.

258. The method of computing it consists in finding, as in simple interest, what is due on a specified amount, added to that amount, and then annually increased by the interest of the preceding year. Method of computation.

EXAMPLE 1.—What will be the compound interest of $500, for 2 years, at 7 per cent.? *Ans.* $72.45

OPERATION.

$500.00	principal 1st yr.
35.00	int. for 1st yr., 500×7=35.
535.00	principal 2d year.
37.45	int. for 2d yr., 535×7=37.45.
572.45	amount in 2yr.
500.00	deduct first principal.
$72.45	amount of interest.

Note.—It will be seen that the whole amount of compound interest is found by taking the first principal from the last sum of principal and interest.

2. What is the compound interest on $1000, for 3 years, at 7 per cent.? *Ans.* $225.043.
3. What will be the amount of $5000, at compound interest, 7 per-cent., for 4yr. 10m.? *Ans.* $6911.26.
4. What is the compound interest of $250, for 2yr., at 8 per cent.? *Ans.* $41.60
5. What is the compound interest of $939.64, for 3yr., at 7 per cent.? *Ans.* $211.45.
6. What is the compound interest of £50 9s. 6d., for 2yr., at 5 per cent.?

Note.—Reduce shillings and pence to decimals of a pound (Art. 214, 14 Ex).

An expeditious way to calculate compound interest is afforded by this

TABLE,

Showing the amount of $1, £1, &c., interest compounded annually at 4, 5, 6, 7 and 8 per cent., from 1 to 20 years.

Years	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	Years
1	1.040000	1.050000	1.060000	1.070000	1.080000	1
2	1.081600	1.102500	1.123600	1.144900	1.166400	2
3	1.124864	1.157625	1.191016	1.225043	1.259712	3
4	1.169859—	1.215506+	1.262477—	1.310796+	1.360489—	4
5	1.216653—	1.276282—	1.338226—	1.402552—	1.469328+	5
6	1.265319+	1.340096—	1.418519+	1.500730+	1.586874+	6
7	1.315932—	1.407100+	1.503630+	1.605781+	1.713824+	7
8	1.368569+	1.477455+	1.593848+	1.718186+	1.850930+	8
9	1.423312—	1.551328+	1.689479—	1.838459+	1.999005—	9
10	1.480244+	1.628895—	1.790848—	1.967151+	2.158925—	10
11	1.539454+	1.710339+	1.898299—	2.104852—	2.331639—	11
12	1.601032+	1.795856+	2.012196+	2.252192—	2.518170+	12
13	1.665074—	1.885649+	2.132928+	2.409845+	2.719624—	13
14	1.731676+	1.979932—	2.260904—	2.578534+	2.937194—	14
15	1.800944—	2.078928+	2.396558+	2.759032—	3.172169+	15
16	1.872981+	2.182875—	2.540352—	2.952164—	3.425943—	16
17	1.947900+	2.292018+	2.692773—	3.158815+	3.700018+	17
18	2.025817—	2.406619+	2.854339+	3.379932+	3.996019+	18
19	2.106849+	2.526950+	3.025600—	3.616528—	4.315701+	19
20	2.191123+	2.653298—	3.207135+	3.869684+	4.660957+	20

7. What is the compound interest of $17.25, for 2yr. and 7m., at 5 per cent.?

Note.—From the table take the amount of $1, for 2 years, at 5 per cent., and compute the interest on it, for 7 months, as in simple interest. Add this to the amount for 2 years, and we have the interest of $1 for 2yr. 7m. Multiply this by $17.25 for answer. To find the interest, subtract the principal from amount.

8. What is the compound interest of $300, for 5yr 8m. 15d., at 6 per cent.? *Ans.* $171.59.

9. What is the compound interest of $600, at 6 per cent, per annum, for 20yr.?

OPERATION.

$3.207135 = int. of $1 for 20yr
600 principal.

$1924.281000 int. of $600 for 20yr

Here, counting off 4 figures on the left for dollars= to the unit-dollars-figures in the multiplicand, and the three of the multiplier, 600, we have the answer, $1924.28.

10. What is the compound interest of $950, for 3yr. 6m. 10d., at 7 per cent.?

DISCOUNT.

Discount defined. **259.** *Discount* is an amount allowed for the cash payment of a bill, or for the settlement of a debt, before its unexpired term of credit, or the sum charged by a bank for money loaned on a note.

The per centage. **260.** The allowance is called per centage, and its rate is according to agreement. When no rate has been named, it is customary to deduct or discount the interest.

To find the amount of discount. **261.** Discount is found by subtracting the present value from the amount due.

EXAMPLE 1.—What is the present value of $100, payable at the expiration of a year, without interest?

Ans. $93.45

OPERATION.	EXPLANATION.
1.07)100.00(93.45+ 963 —— 370 321 —— 490 428 —— 620 535 —— 85	We divide the given sum by $1, with its interest for the time; in this example, 7 per cent.

2. What is the present value of $756, at 6 per cent., payable in 1yr. 4m.?

3. What is the value of $590, due 1yr. hence, at 8 per cent.?

4. What is the discount to be made on a note for $437, which has an unexpired term of 4m. 15d.?

5. What is the present value of a note for $350, payable in 90 days, which has been discounted for me at the Bank of the State? *Ans.* $343.68.

REMARK.—Discount is always required by a bank in advance; thus, in having a note for $350 discounted as above, the sum received would be the answer to that example.

Banks allow a time of notification for payment of a discounted note, of 3 days, called days of grace; in the discount, these are reckoned. Days of grace.

6. What is the bank discount of a note of $1000, payable in 60 days, at 6 per cent. interest? *Ans.* $10.50.

7. My factor having sold for me 30 bags Sea Island cotton, received a note from the buyer for $3456, at 30d.; this note being discounted by the Charleston Bank, at 7 per cent., what amount was credited to my account? *Ans.* $3393.50.

8. What is the present value of a note of $450, payable in 90d. at the bank, at 7 per cent.?

COMMISSION.

262. *Commission* is a certain per centage charged by a factor, broker, or general agent for the transaction of mercantile operations. Commission defined.

263. The commission of a factor is usually for the sale of cotton, rice, and other agricultural products; of a broker, for the sale of merchandize, stocks, uncurrent money, bills of exchange, etc.; of a general agent, for the disposal of various kinds or a particular class of articles. For what sales the commission is paid.

264. The commission is usually at the rate of 2½ per cent. Occasionally it is more or less. The usual rate.

EXAMPLE 1.—A factor sells 25 bales of cotton at $100 per bale: what is his commission at $2\frac{1}{2}$ per cent.? *Ans.* $62.50.

To find the answer.

OPERATION.

$$25\times100=2500$$
$$2\tfrac{1}{2}$$
$$5000$$
$$\text{Mult. by } \tfrac{1}{2} \quad 1250$$
$$62.50$$

Multiplying by ½.

REMARK.—The answer is found by multiplying the amount bought or sold by the number expressing the per centage. To multiply by $\frac{1}{2}$ is to take $\frac{1}{2}$ the amount of multiplicand.

2. A factor sells 3000 bushels of rough rice, at 90 cents per bushel: what is his commission at $2\frac{1}{2}$ per cent.? *Ans.* $67.50.

3. A broker has $9280 to be invested in the Charleston and Savannah Railroad Stocks, which are 15 per cent. above par, or equal value; the broker is to receive 1 per cent.: how many shares of $100 each can be purchased; and what is the commission?

OPERATION.

1.16)9280

$8000. quotient, or 80 shares.

EXPLANATION.

The premium being 15 per cent. and the commission 1 per cent., each dollar of par value costs $1, besides the premium and commission=$1.16. Hence, dividing 9280 by this, we find the amount purchased will be as many dollars as the larger sum contains the less.

Stock at par value; Above par; Below par; Market value.

When stock is rated at its original cost, it is said to be at its par value; when it is rated higher, it is said to be above par, and is at a premium; when lower, below par, and at a discount. Its commercial or transferable worth is known as its market value.

4. A broker invests for a person $5000 in certain stocks, which are 10 per cent. below par, and charges 2 per cent. commission: what is the sum invested? *Ans.* $5434.78.

Note.—As the stock is at a discount, the $5000 must be divided by 90+2=92.

5. A commission merchant has sold for a Southern factory 25 bales of kerseys, for $2550: what is the amount of his commission, at $2\frac{1}{2}$ per cent.; and what is the amount to be paid the owners?

An auctioneer sells at public sale 20 casks bacon, ar average price of \$56 per cask: what is his mission at $2\frac{1}{2}$ per cent, and what is due his loyer?

SIMPLE FELLOWSHIP

35. *Fellowship* or Partnership is an association or nection formed for commercial or trading purposes. Fellowship defined.

36. The money invested by the individuals so ciated or connected is, commercially, the stock apital. The capital invested.

37. The gain resulting is called the profits; the eased value of the capital, through failures or ous mercantile casualties, is known as the loss. The profits. The loss.

38. The profits shared by the persons connected called profit dividends; the losses, loss dividends. Profit dividends; Loss dividends.

XAMPLE 1.—Two persons form a business conion; one invests \$500, the other \$700, and they \$480: how is the gain to be divided?

EMARK.—Since one invests $\frac{5}{12}$, he is to have $\frac{5}{12}$ ne gain, which is \$480=\$200; and, similarly, the r is to have $\frac{7}{12}$=\$280. We, simply to find loss gain, multiply the whole gain or loss by each of person's or company's fractional part of the k. The method to know a share of gain or loss.

Three merchants form a partnership; the first ishes \$3000, the second \$5000, and the third 0; their gain was \$3000: what were the divi-s? *Ans.* The 1st., \$600; 2d., \$1000; 3d., \$1400.

Three merchants, who had severally furnished 0, \$5000, and \$7000, lost by a failure \$800: what of the loss belongs to each, and what amount? *Ans.* $\frac{1}{5}$, $\frac{1}{3}$, $\frac{7}{15}$; \$120, \$200, \$280.

A, B and C freighted a ship with 270 tons. A's ment was 96 tons, B's 72, C's 102. In a storm, ons were thrown overboard: what was the loss ton, and how many tons did each lose? *Ans.* $\frac{1}{3}$ loss on each ton, and A's loss 32, B's 24, C's 34.

5. A ship, valued at $25,200, was lost at sea; there was an insurance on her of $18,000: what was the loss to the owner A, whose investment was to the value of $\frac{1}{3}$; to B, whose was $\frac{1}{2}$; to C, whose was $\frac{1}{6}$?

Ans. A, $2400; B, $3600; C, $1200.

6. A person, failing in business, owes A, $350; B, $1000; C, $1200; D, $420; E, $85; F, $40; G, $20; he has, to meet these amounts, $1557.50: what will be each creditor's share?

7. If a town raise a tax of $1920, and all the property be valued at $64,000, what will it be on $1? what will a person's tax be, whose property is appraised at $1200? *Ans.* .03 on a dollar; $36.

Assessment of taxes. Capitation tax.

When taxes are assessed, an inventory of all real and personal property of the whole town is first taken; and when there is the capitation or poll tax, that of each one subject to it is put down. As the polls are specifically rated, that tax is to be first subtracted from the whole tax, and the remainder to be assessed on the property. To find the amount, each individual is to be taxed for his property, we find how much the remainder of the whole tax is on $1, and multiply his inventory by it.

General tax. Special tax.

In some States there is no capitation tax, and the sum to be raised for the expenses of the Government is collected from each individual, in proportion to his property. In South Carolina, this is on land and negroes, and is called the general tax. Apart from this, there are, in incorporated towns, special taxes on houses, servants, carriages, horses, etc.

8. In a certain town where there is the capitation tax, the amount to be raised is $5999. The real estate is valued at $500,000 and the personal at $300,000. There are 666 taxable polls, each of which is assessed $1.50: what is the tax of A, whose real estate is valued at $4000 and his personal property at $8000, and who pays one capitation tax?

Ans. $76.50.

DOUBLE FELLOWSHIP.

269. *Double Fellowship* varies from Simple Fellow- Double fellowship defined
p in the investment of shares in a company for
equal terms of capital and time.

EXAMPLE 1.—A and B form a partnership for the
insaction of certain business. A puts in \$300 for
and B \$400 for 7m.; their gain is \$156: what is the
are of each? A's \$300 for 8m =2400 for 1m.
B's \$400 for 7m.=2800 for 1m.

5200 for 1m.

Here we have the joint stock of \$5200 for 1 month,
which A puts in \$2400 and B \$2800; thus, A is to
ve $\frac{2400}{5200}=\frac{6}{13}$ of the gain, and B $\frac{2800}{5200}=\frac{7}{13}$; or, A $\frac{6}{13}$
\$156=\$72, and B $\frac{7}{13}$ of \$156=\$84.

Note.—Each partner's stock is multiplied by the
ne of its engagement.

2. Three merchants, A, B and C, enter into part-
rship; A puts in \$60 for 4m., B \$50 for 10m., and
\$80 for 12m.; they lose \$50: what is the share of
is to each? *Ans.* A, \$7.05; B, \$14.70; C, \$28.23.

3. Four traders form a connection; A puts in \$400
. 5m.; B, \$600 for 7m.; C, \$960 for 8m.; D, \$1200
r 9m; they lost \$750: what loss did each sustain?
Ans. A, \$60.77; B, \$127.63; C, \$233.38; D, 328.20.

4. A commenced business November 1, with a
pital of \$3400. On the 1st February he associated
th him B, who invested his capital of \$2600. In a
elvemonths' time, they had gained \$750: what is
e share of each? *Ans.* A's, \$476.63; B's, \$273.36.

5. Two merchants having entered into partnership
r 16m., invested as follows: A, at first, to the
nount of \$1690, and, at the end of 9m, \$900 more;
at first, \$650, and, at the end of 6m., withdrew
50; they gained \$500: what was each one's part?

6. On the 1st January, A commenced business with
40; on the 1st February after, he associated with
m B, who put in \$660; on the 1st June C was ad-
itted to the firm, with a capital of \$1800; at the
d of the year, they had gained \$992: what was
e share of each?

INSURANCE.

Insurance defined.

270. *Insurance* is the engagement of a company to *protect*, for a specified time, a certain property from loss by fire or other casualty.

The policy.

271. The written contract assuring such protection is called the *policy*.

The underwriters.

272. The persons pledged to its performance are known as *underwriters*.

The premium.

273. The sum paid for such risk or service is termed the *premium*.

Fire, marine, and life insurance cover all risks.

274. Fire, marine and life insurance embrace the risks for which policies are given.

Its per centage various.

275. The per centage for which insurance against fire is effected, varies according to the nature of the property or its locality; against marine or sea disasters, according to the strength of the vessel, the voyage, or other circumstances in such connection; against the loss of life, according to the age, health, or exposure to sickness or danger of the individual.

EXAMPLE 1.—What would be the premium for the insurance of a house valued at $5000, against loss by fire, for 1yr., at ½ per cent.? (5000÷2=25.)

Ans. $25.

Note.—We simply divide by the denominator of the fractional per centage, ½, and as it is a per cent. diviisor, pont off two figures in the result for cents.

2. What would be the premium for insuring a ship and cargo valued at $37,500, from Charleston to Liverpool, at 3½ per cent.? *Ans.* $1312.50.

OPERATION.	EXPLANATION.
37500 3½ ——— 112500 18750 ——— 1312.50	We multiply by 3, and to its result add the result of the multiplication by ½ (37500÷2=18750).

3. What is the insurance on a store and goods valued at $15,000, at 2¼ per cent.? *Ans.* $337.50.

4. A merchant owns $\frac{3}{4}$ of a ship, valued at \$24,000 and insures his part at $2\frac{1}{2}$ per cent.: what does he pay for the policy? *Ans.* \$450.

5. A shipment of goods from Liverpool, valued at £534 10s. 6d., has been insured at $1\frac{3}{4}$ per cent.: including cost of policy paper, \$1.25, what must be paid in American money, calculating the pound at \$4.87? *Ans.* \$46.80.

6. I have effected an insurance on a friend's house in Augusta, Ga., for \$3000, at $\frac{5}{8}$ per cent.; furniture \$1500, at $2\frac{1}{2}$ per cent.; stable, horses and carriage \$2000, at 3 per cent.: what is the insurance? *Ans.* \$168.75.

7. A vessel and cargo worth \$65,000, are damaged to the amount of 20 per cent., and there is an insurance of 50 per cent. on the loss: how much will the owner receive? *Ans.* \$6500.

8. What will be the annual premium for insuring \$5000, for 12 years, the life of a man 40 years old, at the premium of \$1.09 per cent.?

PROFIT AND LOSS.

276. *Profit and Loss* is the name applied to the process used in mercantile transactions to ascertain the favorable or unfavorable result of a financial operation. It is a combination of rules already known, and very important in this connection as showing a method practically valuable to assure a certain per centage of profit or loss. Profit and loss defined.

EXAMPLE 1.—Bought 300 lbs. of sugar at 9 cents per lb., and sold the same for $12\frac{1}{2}$ cents per lb.: what was the profit? *Ans.* \$10.50.

OPERATION.

Purchase.	*Sale.*
300	300
9	$12\frac{1}{2}$
27.00	3600
	150
	37.50
	27.00
	10.50

EXPLANATION.

Here, after the multiplications of purchase and sale, the difference is found between the two for answer.

2. Bought 250 yds. of cloth at $225. per yd.: what must it be sold for to gain 7 per cent.? *Ans.* $601.87

OPERATION.

```
     250
     225
    1250
    500
   500
   $562.50 =the cost.
         7 gain proposed.
  $39.37.50
 $562.50
 $601.87
```

EXPLANATION.

Having found the cost, it is necessary to multiply that amount by the gain proposed, and we find the answer $39.37; adding this to the cost, we obtain for answer as above.

Repetition of the direction to point off certain figures.

Note.—In the multiplication of dollars and cents by cents, let it be remembered that 4 figures must be pointed off. The two at the extreme right are not in these calculations usually considered. The next two are cents; those at the left of the point are dollars.

3. Bought 14 bbl. flour for $100, and sold the same at $8 per bbl: what was the gain? *Ans.* $12.

4. Bought 100 bushels corn for 75 cents per bushel, and sold it for $60: what was the loss?

5. Bought 38 bushels sweet potatoes at $62\frac{1}{2}$ per bushel, and sold them at 75 cents per bushel: what was the gain?

6. Bought 60 hhds. of wine at $39 a hhd., and sold the same at the rate of $50 a hhd., but discounted 3 per cent for cash: what was the profit?

7 Bought 525 bbls. of flour, at $5.35 per bbl., and sold them at $6.25 per bbl., with a discount of 2 per cent. for cash: what was the profit?

8. Bought 7 hhds. of bacon for $460, and sold them for $500: what was the gain per cent., or per 100?

Ans. $$8\frac{16}{23}$.

Note.—This can be solved by simple proportion, and is the same as if the question had been thus put: If $460.00 gain $40, what will $100 gain?

```
        460 : 40 : 100 :
                    40=the difference between 460
                                       [and 500.
              460)4000(Ans. $8 16/23.
                  3680
 Reduced by  10) 320
                 460 or 32/46, or 16/23.
```

9. Bought 4hhds. of sugar, each weighing 250lbs., for '0.00, and sold them again at 8 cents per lb.: how much as the profit per cent.?

Note.—Find the net profit on the whole, then the per nt, by proportion, as in the 8th example.

10. What is the profit, per cent., on a yard of cloth, ought at \$4 and sold at \$5?

Cost : Gain :: Par value : Gain per cent.
\$4 : \$1 :: 100per ct. : 25 per cent. *Ans.*

Note.—The par value of an article is its first cost. Par value defined.

REMARK.—Here, we ascertain the difference between the purchase, price and sale, and say, as the purchase rice, or cost, is to the total gain, or loss, so is 100 per ent., or par value, to the gain or loss per cent. A proportional statement

11. Bought 500bbl. pork at \$12 per bbl., and gave ote at 6m., in settlement: what was the profit, the same aving been sold at \$15 per bbl., cash?

12. A merchant purchased 20 chests of green tea at 40 per chest, and settled for them with a note, payable in m. He afterward sold them for \$46 per chest, at 30 ays: what did he gain, if the bank discount were eckoned at 7 per cent.?

REMARK.—Find the cash cost of the 20 chests, by educting $3\frac{1}{2}$ per cent. ($7\div2=3\frac{1}{2}$) from their cost, at \$40. ind, too, the cash price of sale by deducting $\frac{7}{12}$ per cent. $12\div7=\frac{7}{12}$ or 1m. int.) from the price of 20 chests$\times$\$40. he difference between the cash cost and the cash price of ale is the profit.

13. A merchant bought 25hhd. of molasses at \$30 per hd., and gave his note at 90 days. A month later, he old them at \$35 per hhd., and received a note payable in m.: what was the profit, the bank discount being at the ate of 6 per cent.?

REMARK.—Here, 2 per cent. is to be deducted from he price of the sale, as the goods were 1 month on hand, nd were not paid for until 6m. had expired; making in ll 7m. interest, at 6 per cent., which, for $7m.=3\frac{1}{2}$ per cent.

14. A grocer bought 75bbl. of cider at \$2.25 per barrel, and gave his note, due in 6m. Two months later, he sold them at \$3 a barrel, on a credit of 4m.: what was his profit, the bank discount being at the rate of 6 per cent.?

15. Bought an invoice of books for \$700: how much must the same be sold for, in order to gain 25 per cent.?
Ans. \$875.

The purchase price given to find selling price.

OPERATION.

```
 700
  25
 ----
 3500
1400
 ----
175.00
700.
 ----
$875.
```

EXPLANATION.

We here multiply the purchase price by the per cent., and add the result to the cost. The amount would have been subtracted had the per centage been loss.

16. What must 15 pipes of wine be sold for, which cost \$55 a pipe, to gain 12 per cent. on the cost?

17 What must 35 bags of cotton bring, that cost \$75 a bag, to gain 15 per cent.?

18. Bought an invoice of figs at $12\frac{1}{2}$ cents per lb.: proving injured, what must they be sold for to lose 10 per cent.? *Ans.* 11 cents and $2\frac{1}{2}$ mills.

Note.—This is easily solved by proportion; thus,

A proposition in profit and loss solved.

OPERATION.

```
100 : 90 :: 12½
              90
            ----
            1080
              45
            ----
       100)1125 (11¼ or 11c. 2½ mills.
            100
            ----
             125
             100
             ---
Reduce by 25) 25/100 = ¼
```

EXPLANATION.

Here, we say, as 100 is to 90 (=100—10) so is the purchase price to the selling price.

19. A trader sold apples at \$1.50 per bbl., and lost 10 per cent. by the sale: what was the cost? *Ans.* $\$1.66\frac{2}{3}$.

OPERATION.

```
90 : 100 :: 150 :
             100
            ----
        90)15000(166 4/9
           90
           ---
           600
           540
           ---
            600
            540
            ---
             60
```

20. A merchant bought a piece of velvet at \$5 per yard, but it being damaged, he proposed to sell it, so as to lose 20 per cent.: how must it be marked, so that 10 per cent. be deducted from the price at which he had designed to sell it? 90 : 80 :: 5 : *Ans.*

REMARK.—Here, it is said, as 100, diminished by the per cent. to be deducted, is to 100, increased by the per cent. to be gained, or diminished by the per cent. to be lost, so is the cost to the proposed price of sale.

21. Bought a mule for \$175 : what, so as to lose 5 per cent. on the cost, shall I ask for it, so that I may diminish the price 20 per cent.?

EQUATION OF PAYMENTS.

277. *Equation of Payments* is an operation to discover the mean time for settlement of debts incurred at previous dates, so that neither party interested shall suffer loss. Equation of payments defined.

EXAMPLE 1.—Supposing a person owes me \$100, due in 30 days; \$200, due in 25 days; and \$150, due in 10 days, but wishes to settle all at once: what is the mean time of payment; or, equivalently, how long is he to keep the money if he wishes to pay all at once?

OPERATION.

$100	$200	$150
30	25	10
3000	1000	1500
	400	
	5000	

These added =9500, which, divided by 450, the amount of the different debits, gives 21 nearly, and is the time sought.

To equate payments.

REMARK.—From this, we learn to multiply each debt by the time at which it falls due, and divide the product by the indebtednesses, for the mean time.

2. I owe $500, one half payable to-day and the remainder in 8m.: when is the equated day for settlement?

Ans. 4m.

3. A owes B three different sums of money—$150 due in 3m., $240 in 60 days, and $300 in 4m.: what is the mean time of payment?

4. A owes B $500 due in 3m., $350 in 2m., and $200 in 1m.: what is the mean time of payment?

5. A owes B four different sums of money, as follows: $1000 due in 3m., $500 in 6m., $400 in 5m., and $300 in 2m.: as he wishes to settle these at one time, what is the equated date?

6. A owes B $400, which is, by agreement, to be paid in 4 equal instalments; the first in 30 days; the second in 60 days; the third in 90 days, and the fourth in 4m. He wishes to settle all at one date: what is the equated time?

7 A merchant purchased a certain commodity for which he gave his notes; one of $300, payable in 3m., and another of $350, payable in 6m. One month after the purchase, he proposes to give a single note for the whole amount: for what time must the note be written?

REMARK.—The first note having 2m. of unexpired time, and the second 5m., multiply $300 by 60d., and $350 by 150d. (5m.)

8. When is the mean time for payment of $400 due in 6m., $500 in 8m., and $1000 in 12m.? *Ans.* $9\frac{13}{19}$.

9. Two merchants had the following business transactions: A purchased of B a bill of goods, May 15, 1860, on 3m. credit, for $200; May 1, 1860, on 4m. credit, for $600; and B purchased of A a bill of goods, May 15, 1860, on 3m. credit, for $300; June 14, 1860, on 4m. credit, for $900: when does the balance owned by B fall due?

As A's debt is found to be due August 20, and B's September 29, 40 days after A's is due, the question is, when is B to pay the balance of $400?

REMARK.—Had A and B each paid their debts when the time was up, the question would have been plain; but as the account is to be settled by B's paying the balance of $400, he can keep that sum long enough, after September 29, to equal A's holding $800 40 days after it was due; as $800 for 40 days equals $400 for 80 days=32,000, and this, divided by 400=80. Hence, 80 days from September 29, is the mean time.

10. Two houses in Savannah had the following transactions: A purchased a bill of B, January 7, 1861, 1m. credit, for $800; February 7 1861, 2m. credit, for $566.66⅔; February 7, 1861, 3m. credit, for [illegible]. B purchased of A a bill, January 18, 1861, 2m. credit, for $200; February 26, 1861, 4m. credit, for $1200; March 1, 1861, 3m. credit, for $800; March 26, 1861, 3m. credit, for $800: when shall B pay to A the balance?

Ans. January 27, 1862.

BARTER.

278. *Barter* is the exchange of commercial values.

279. Though separately placed, it rightfully belongs to the solution of questions by analysis.

Its questions are those of analysis.

EXAMPLE 1.—How many pounds of butter, at 22 cents per lb., must be given for a chest of tea, containing 75lbs., at 80 cents per lb.?

OPERATION.

```
        75
         80
      -----
22)60.00(272 16/22 = 8/11.
   44
   ---
   160
   154
   ----
     60
     44
     --
     16
```

EXPLANATION.

The value of the tea being ascertained, the quantity of butter to pay for it is found by dividing the tea's value by the price per lb. of the butter.

2. A has 300yd. of cotton bagging worth 30 cents per yard, which he wishes to exchange for corn at 75 cents per bushel: how many bushels can he have? *Ans.* 120.

3. A flour merchant had 200bbl. of flour, valued at $10.50 per barrel, for which a grocer gave him, in money, $1090, and the balance in Cuba molasses, at 20 cents per gallon: how many hhd. of molasses did he receive?

4. What number of barrels of apples, at $1.20 per barrel, will purchase 20 cords of wood, at $3.50 per cord?

5. A merchant exchanged 1 case, 50pcs.=1500yd. calico, worth 9 cents per yd., for 60pcs.=1800yd. long cloth, at 10½ cents per yd.: what was the difference that he had to pay?

6. A trader received in exchange for 300pr. brogans, valued at 95 cents per pair, 60 hides, at $1.62½ each: how much was the balance in his favor?

PRACTICE.

Practice defined.

280. *Practice* is a process for solving questions, by substituting for large multiplications and divisions aliquot parts, such as ½, ⅓, ¼, etc.

A contraction of previous rules.

281. It is mainly a contracted form of rules already considered, but especially of proportion. As its name indicates, the method of solving questions proper to it can only be acquired by constant use.

EXAMPLE 1.—What is the cost of 48yd. of satinet, at 75 cents per yd.?

OPERATION.

$\frac{1}{2}$ dollar)	48
$\frac{1}{4}$ dollar is $=\frac{1}{2}$ of $\frac{1}{2}$	24
	12
	\$36 *Ans.*

EXPLANATION.

We here first find, by dividing the yds. by $\frac{1}{2}$ dollar, the value at $\frac{1}{2}$ dollar a yard; and then, taking $\frac{1}{2}$ of that answer, and adding it to the sum of the yd. at $\frac{1}{2}$ dollar, find the true result. ($\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ and $\frac{3}{4}$ dollar$=.75$.)

2. What is the value of 80yd. of furniture chintz, at 20 cents a yard? ($\frac{1}{5}$ dollar$=20$ cents.)

3. What is the price of 36 barrels of ale, at \$3.75 per barrel?

OPERATION.

	36
	3
	108
$\frac{1}{2}$ of $36=$	18
$\frac{1}{2}$ of $18=$	9
	\$135

EXPLANATION.

We here first multiply by 3 for the value at \$3; then, we take $\frac{1}{2}$ of 36, for the 50 cents' value; and, last, $\frac{1}{2}$ of 18, which is $\frac{1}{2}$ the value of 50 cents.

4. What is the value of 42yd. of woollen, at 25 cents per yd.?

5. What will 5cwt. 3qr. 16lb. cost, at \$4.20 per cwt.?
Ans. \$24.82.

6. What is the price of 66yd. of ladies cloth, at $62\frac{1}{2}$ cents per yd.?

7. What will 250yd. of muslin cost, at $37\frac{1}{2}$ cents per yard?

8. What will 130yd. cassimere cost, at $87\frac{1}{2}$ cents per yard?

9. What is the value of 160yd. of cotton flannel, at $12\frac{1}{2}$ cents per yd.?

10. What is the value of 140yd. of broadcloth, at £1 12s. 6d. per yd.?

OPERATION.

	£.	s.
For £1	$=140$	00
For 10s. ($\frac{1}{2}$ £)	70	00
For 2s. ($\frac{1}{5}$ 10s.)	14	00
For 6d. ($\frac{1}{4}$ 2s.)	3	10
Ans.	£227	10

EXPLANATION.

At £1 per yd., 140yd.= £140. As 10 shillings= £$\frac{1}{2}$, we take for the 10s. $\frac{1}{2}$ £140= £70. As 2 shillings $=\frac{1}{5}$ 10 shillings, we take $\frac{1}{5}$ £70= £14, which is the value for 10 shillings. As

the 6d.=$\frac{1}{2}$ of 1. or $\frac{1}{4}$ of 2 shillings, we take $\frac{1}{4}$ £14=£3 10s. These values added give the result sought.

11. What will 36 bushels of wheat cost, at 7s. 6d. per bushel? *Ans.* £13 10s.

12. What is the cost of 3cwt., 3qrs. 21lbs., at £3 15s. 6d. per cwt.?

OPERATION.

		£	s.	d.	
		£3	15s.	6d.	
				3	
		11	6	6	cost of 3cwt.
$\frac{1}{6}$ 3cwt.	=	1	17	9	cost of 2qrs.
$\frac{1}{2}$ 2qrs.	=		18	10$\frac{1}{2}$	cost of 1qr.
$\frac{1}{2}$ 1qr.	=		9	5$\frac{1}{4}$	cost of 12$\frac{1}{2}$lbs.*
$\frac{1}{2}$ 12$\frac{1}{2}$lbs.	=		4	8$\frac{1}{2}$	cost of 6$\frac{1}{4}$lbs.
Ans.		£14	17s.	3$\frac{1}{4}$d.	

13. What will 18yds. cloth cost, at 14s. 6d. per yd.?

14. What will 24yds. cloth cost, at 12s. per yd.?

15. What will 2cwt., 2qrs. 18lbs. sugar cost, at $7.50 per cwt.?

16. What will 6gals. 3qts. oil cost, at $2.25 per gal.?

EXCHANGE.

Exchange defined.

282. *Exchange* is a convenient process for the transmission of funds to persons residing or travelling abroad; or to parties living in other places of the same country.

How exchange is effected.

283. Such transmission is effected through the medium of a draft or bill, which is commonly an order from a bank to an agent commercially in its connection; or from a banker or broker, whose business is the transfer of such paper to some house with which he has financial understanding.

An inland and foreign bill.

284. When the parties are residents in the same country, such draft is called an inland bill; otherwise, a foreign bill.

* 1 quarter=25lbs.

235. The value of the pound sterling (Art. 101), is $4.44,4 (=$\frac{40}{9}$), at par. But in settling accounts between this country and Great Britain, or in the purchase of bills of exchange, it is seldom that British currency is to be had at that rate. Usually it is at an advance of 5 to 20 per cent. Sometimes it is at a discount, or below par, or below the intrinsic value of $4.44.4 mills. *The value of the pound sterling.* *Exchange variable.*

236. Agreeably to these fluctuations, exchange is said to be at par, at a premium, or at a discount. *Be at par, premium, or discount.*

EXAMPLE 1.—A merchant in Charleston wishes to pay for a bill of sugar, due in New Orleans, $2550. He finds that exchange is at 2 per cent. premium: what must he pay for a draft? *Ans.* 2601.

OPERATION.

```
   2550
      2
  ------
 $51.00 prem.
 $2550.  bill.
 -------
 $2601.  draft.
```

2. A company in Savannah wish to remit to Galveston, Texas, $500, and find that exchange is 1 per cent. below par: what must they pay for their bill? *Ans.* $495.

3. A company trading in Mississippi, propose to remit to Richmond, Va., $5900, but find a bill cannot be purchased under 2½ per cent. premium: what will the per centage be?

4. A banker in Charleston, South Carolina, wishes to send to Liverpool a bill of exchange for £112 15s. 6d.: what must he pay for the bill when exchange is 9½ per cent. premium? *Ans.* £123 4s.

OPERATION.

£112 15s. 6d.=£112.775 (Art 214, 14 Ex.) add 9½ per ct.
10.713

£123.488×40 and÷9=$548.83.

REMARK.—In bills of exchange on England, the £ sterling is still valued at 4\frac{4}{9}$=$4.44, instead of its present worth, $4.84: hence, £1=$4.44 *How the £ is valued.*

Add 9 per cent. .399

$4.83,9 which expresses the value.

Note.—Pounds and decimals of a pound are reduced as above, to dollars, by multiplying by 40 and dividing by 9.

It is convenient to be able to exchange, expeditiously, British into American currency. The following method will be found useful to facilitate such computation:

5. Change 19s. 4½d. to a decimal fraction of a pound.

An easy way to change British into American money.

OPERATION.

19s. 4½d.
0.9
18
5
1
———
Ans. 0.969

EXPLANATION.

We take half the even number of shillings for a first decimal figure; in this example, 9; we then change the remaining pence and farthings to farthings, which are the second and third decimal figures; in this example, 18; then, as the number of shillings was odd, we add 5 in the second place, and, as the number of farthings exceed 12, we add 1 to the third figure.

Note.—By multiplying decimal 0.969
by 40
———
and dividing by 9)38760
———
we have answer in $, cents and mills $4.30,6m.

6. Change 15s. 6d. to a decimal fraction of a pound.
Ans. 0.775=$3.44.

7. Change 13s. 5d. to a decimal fraction of a pound.

8. Change 17s. 3½d. to a decimal fraction of a pound.

9. Change 16s. 2¼d. to a decimal fraction of a pound.
Ans. 0.89=$3.95,5m.

OPERATION.

16s. 2¼d.
0.8
9
———
0.89

Note.—In this example, the shillings being even and the farthings less than 12, the addition of the 5 and the 1 are not required.

10. Change 10s. to a decimal fraction of a pound.
Ans. 0.5=$2.22,2m.

11. Change 16s. 7d. to a decimal fraction of a pound.

12. I wish to remit to London a bill of exchange for £90: how much must be paid for the bill, when exchange is at $9\frac{1}{2}$ per cent. premium?

OPERATION.

$$\begin{array}{lr} & £90 \\ \text{Add } 9\frac{1}{2} & 8.55 \\ \hline & £98.55 \end{array} = \text{Am. money, } \$438.$$

13. Imported from England a bill of goods amounting to £75 6s. 3d.: what, in American money, will that be, when exchange is 5 per cent.?

14. My factors in Mobile shipped to Liverpool 55 bales of cotton, weighing 22,500 pounds; it was sold at 15d. (1s. 3d.) per lb. The freight and other charges amounted to £150 15s. Having sold the bill of exchange, which was received in payment, at $9\frac{3}{4}$ per cent. premium, how much would the amount be, in American money?

Ans. $6724.44.

15. I have requested my factors to transmit to my banker, in Paris, a bill of exchange for 4644 francs: at the par of exchange, what will be the amount in American money?

REMARK.—French currency is computed in francs and centimes; the franc being equal to $18\frac{3}{4}$ cents, American money, and each centime $\frac{1}{100}$ part of a franc. To change francs to American money, at the par of exchange, we multiply the number of francs by $18\frac{3}{4}$, and divide by 100: or, what is equivalently such division, we point off 2 decimals and have the answer in dollars and cents.

French currency.

French exchanged to American money.

16. Change 25 francs to American money, at the par of exchange.

OPERATION.

$$\begin{array}{r} 25 \\ 18\frac{3}{4} \\ \hline 200 \\ 25 \\ \left.\begin{array}{r} 12\frac{1}{2} \\ 6\frac{1}{4} \end{array}\right\} \text{the multiplication by } \tfrac{3}{4}. \\ \hline \end{array}$$

Ans. $\$4.68\frac{3}{4}$.

17. Change 325 francs to American money, at the par of exchange.

18. Change 560 francs to American money, at the par of exchange.

19. What will be the value of a bill of exchange on Paris, in American money, for 8550 francs, the rate of premium being 5.12 francs per dollar? *Ans.* \$1685.20.

Note.—Here, the exchange=5fr. 12 centimes, has to be added.

20. What will be the value of a bill of exchange on Paris. in American money, for 1500 francs, there being a discount on French exchange of 5.13 per dollar?

Note.—Here, the exchange has to be subtracted.

To change American to French money. REMARK.—When it is required to change American money to French, the process is reversed.

21. Change \$4.68¾ to French money, at the par of exchange.

22. Change \$870.75 to francs. *Ans.* 4644.

OPERATION.

```
18¾ 870.75
 4       4
—   ———————
75)3483.00(4644fr. Ans.
   300
   ———
    483
    450
    ———
     330
     300
     ———
      300
      300
```

Note.—For the convenience of the division, both divisor and dividend are changed to 4ths. This enables us to avoid the fractional ¾, but does not change the value of the result.

23. Change \$750 to francs, adding premium 5.15.

24. What will be the value of a bill of exchange on Paris, in American money for 8550 francs, the premium being 5.08?

GUAGING.

287. *Guaging* is the process used to find the contents of vessels chiefly of curved form. Guaging defined.

288. Exactness, because of the different curvatures of staves, in finding the interior measurement of such vessels, is impossible; but the approximation to it is sufficiently near to answer all commercial purposes. Exactness impossible.

289. The mean diameter of a cask is found by adding to the head diameter $\frac{2}{3}$ of the difference between the bung and head diameter, or when the staves are not greatly curved, by adding $\frac{6}{10}$. This gives the vessel a cylindrical form. We then multiply the area of the base by the altitude. To find the mean diameter.

290. To find the solid contents in cubic inches, we multiply the square of the mean diameter of the cylindrical vessel by the decimal .7854, and the product by the length. The contents in gallons are found by dividing by 231, which is the number of cubic inches that expresses a gallon of liquid measure. (Art. 90.) To find the solid contents in inches; The contents in gallons.

291. When a vessel is very irregular, or when a cavity not very large is to be measured, the easiest and most accurate way to find the contents is to fill the vessel or cavity with water, and then to measure this put in casks whose dimensions are found as above. When a vessel is very irregular.

EXAMPLE 1.—How many gallons are in a cask whose bung diameter is 40 inches, head diameter 28 inches, and length 48 inches? *Ans.* 211.59gal.

OPERATION.

$40 - 28 = 12$

$\frac{2}{3}\ 12 = 8$

$28 + 8 = 36$

$36 \times 36 = 1296$

$1296 \times 48 \times 34 = 211.59$gal.

EXPLANATION.

Having found the difference of the diameter, we take $\frac{2}{3}$ of it and add to the head diameter. We then multiply the square of the mean diameter (36×36), the length (48), and 34 together, and point off 4 decimal places. This gives the answer in gallons and decimals of a gallon. Explanation of the example.

Contents in gallons found by use of a decimal.

REMARK.—The decimal .7854 divided by 231 (=the cubic inches in a gallon liquid measure), carried to four decimal places, is .0034; and this decimal multiplied by the square of the mean diameter, and by the length of cask, gives the contents in gallons.

2. What are the contents, in wine gallons, of a cask whose length is 36 inches, and whose head and bung diameters are each 16 and 19 inches?

How to measure a curved vessel.

Note.—To measure curved vessels, simply multiply the length by the square of the mean diameter, then by 34, and mark off 4 decimal places.

3. What are the contents, in beer measure, of a cask whose length is 46 inches, and whose head and bung diameters are each 24 and 32 inches?

Note.—In beer measure, the decimal .7854 is divided by 282 cubic inches=the beer gallon. (Art. 94.)

4. What are the contents, in bushels, of a hhd. whose length is 48 inches, and whose head and bung diameters are 34 and 42 inches?

Note.—In dry measure, the bushel equals 2150 cubic inches.

5. What are the contents, in bushels, of a cask whose length is 44 inches, and whose head and bung diameters are 32 and 40 inches?

6. What are the contents, in wine gallons, of a cask whose length is 38 inches, and whose head and bung diameters are each 15 and 18 inches?

7 In a barrel 30 inches deep, and its diameter (one third from the top) 20 inches, how many wine gallons?

$$20\times20=400. \quad 400\times30=12000$$

$$\begin{array}{r} 12000 \\ 34 \\ \hline 48000 \\ 36000 \\ \hline 408000=408\ 10\text{gal.} \end{array}$$

Note.—Multiply the diameter (one third from the top)

by itself, and this by the depth. Then multiply by 34 and cast off 4 figures for decimals.

8. In a barrel 34 inches deep and its diameter 18 inches, how many wine gallons?

9. In a barrel 28 inches deep and its diameter 16 inches, how many wine gallons?

Note.—For the guaging of corn, peas and potatoes, see 60–63 pages.

TONNAGE.

292. The *Tonnage* of a vessel is her measured capacity of freight. **Tonnage defined.**

The quantity that can be carried is estimated by two rules; one known as the carpenter's, the other the government's measures. **Two rules to learn the tonnage.**

EXAMPLE 1.—What is the tonnage of a single decked vessel whose length is 80 feet, breadth 21 feet, and depth 18 feet? *Ans.* $318\frac{6}{19}$ tons.

OPERATION.	EXPLANATION.
21 80 —— 1680 18 —— 13440 1680 —— 95)30240($318\frac{6}{19}$. 285 —— 174 95 —— 790 760 —— 30	We here multiply the length of keel, breadth at main beam, and depth of hold, in feet, together, and divide by 95; the quotient is the number of tons.

To measure a double decked vessel.

Note.—For a double decker take $\frac{1}{2}$ of the breadth at the main beam, for the depth of the hold, and proceed as above.

2. What is the carpenter's tonnage of a single decked vessel whose length is 90 feet, breadth 25 feet, and depth 19 feet?

3. What is the carpenter's tonnage for a double decked vessel whose length is 200 feet and breadth 38 feet?

Ans. 1520 tons.

4. What is the government's tonnage for a vessel of single deck whose length of keel is 80 feet, breadth at main beam 21 feet, and depth 18 feet?

Government measure example.

OPERATION.

$$\begin{array}{r}
80 \\
12\frac{3}{5} \\
\hline
67\frac{2}{5} \\
21 \\
\hline
67 \\
134 \\
8\frac{2}{5} \\
\hline
1415\frac{2}{5} \\
18 \\
\hline
11320 \\
1415 \\
7\frac{1}{5} \\
\hline
95)25477\frac{1}{5}(268\frac{17\frac{1}{5}}{95} \text{ tons.} \\
190 \\
\hline
647 \\
570 \\
\hline
777 \\
760 \\
\hline
17\frac{1}{5}
\end{array}$$

EXPLANATION.

From the length we take $\frac{3}{5}$ of the breadth ($80-12\frac{3}{5}$), and having multiplied the remainder by breadth and depth, divide by 95 for the result required.

How to measure a single decker, government rule.

REMARK.—By government rule, for a single decker, take length, in feet, above the deck, from the fore part of the main stem to the after part of the stern post; the

breadth, at the widest part above the main wales, on the outside; and the depth, from the under side of the deck plank to the ceiling in the hold.

5. What is the government tonnage for a vessel of the same capacity as the one given in 3d example?

REMARK.—To measure a double decked vessel, take the length above the upper deck; for the depth, take ½ the width, and proceed as before directed. To measure a double decker.

6. What is the carpenter's tonnage for a single decker whose length is 100 feet, breadth 25 feet, and depth 20 feet?

7. What is the government's tonnage for the same capacity? *Ans.* 447$\frac{7}{19}$ tons.

ANNUITIES.

293. An *Annuity* is a sum of money payable to a person for a certain term of years; usually, for the life time. Annuity defined.

294. An annuity not paid at the stipulated date is said to be in arrears; when it is not to commence until some future time, it is called a reversionary annuity; but when its payments have commenced, it is said to be in possession. An annuity in arrears; In reversion In possession

295. Annuities are often bought and sold, as other commercial values. Bought and sold.

296. The sum of annuities, such as rents, pensions, salaries, remaining unpaid, with the interest on each, is called the amount of the annuity. The amount of an annuity

297. To find the value of an annuity in arrears observe simply the method to ascertain an amount at interest; but, for an expeditious way to discover the same, we give the following: To find its value.

TABLE,

Table showing certain amounts.

298. *Showing the amount of the annuity of $1, £1, etc., at 4, 5, 6 and 7 per cent. compound interest, for any number of years not exceeding* 20.

Years	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.0000	1.0000	1.0000	1.0000
2	2.0400	2.0500	2.0600	2.0700
3	3.1216	3.1525	3.1836	3.2149
4	4.2464	4.3101	4.3746	4.4399
5	5.4163	5.5256	5.6370	5.7507
6	6.6329	6.8019	6.9753	7.1532
7	7.8982	8.1420	8.3938	8.6540
8	9.2142	9.5491	9.8974	10.2598
9	10.5827	11.0265	11.4913	11.9779
10	12.0061	12.5778	13.1807	13.8164
11	13.4863	14.2067	14.9716	15.7835
12	15.0258	15.9171	16.8699	17.8884
13	16.6268	17.7129	18.8821	20.1406
14	18.2919	19.5986	21.0150	22.5504
15	20.0235	21.5785	23.2759	25.1290
16	21.8245	23.6574	25.6725	27.8880
17	23.6975	25.8403	28.2128	30.8402
18	25.6454	28.1323	30.9056	33.9990
19	27.6712	30.5390	33.7599	37.3789
20	29.7780	33.0659	36.7855	40.9954

EXAMPLE 1.—What is the amount of an annual pension of $500 in arrears for 16 years, at 7 per cent. compound interest? *Ans.* $13,940

Note.—The table shows the 7 per centage of 16 years to be 27.8880; this multipled by the amount, 500, gives 139440000=$13,940.

2. What is the amount of an annual salary of $1500, which has been in arrears for 10 years, at 7 per cent.?

3. What is the present worth of an annual pension of $120, at 6 per cent., to continue 3 years?

Ans. $320.76.

Explanation of method.

EXPLANATION.—We find by the table that $120= $382.03; this divided by the amount of $1, compound interest, gives the quotient of present worth. This is evident, since the quotient, multiplied by the amount of

$1 for 3 years, at compound interest, is $382.03. By reference to the table under the article, Interest, the amount of $1 at compound interest, is seen to be $1.1910; then $382.03 divided by it gives $320.76.

4. What is the present value of an annuity of $300, at 7 per cent., to continue 10 years?

5. What is the value of a pension of $90 per annum, in arrears, for 12 years, at 7 per cent.?

6. What is the value of a ground rent of $200 per annum, for 8 years, at 7 per cent.?

7 For what can I buy with cash, an annuity of $300, to continue 5 years?

8. If you lay up $50 a year from the age of 21 to that of 60, what will be the amount at compound interest?

ALLIGATION.

Alligation defined.

299. *Alligation* is a mercantile usage to procure, by the mixture of simple substances of different qualities, a compound of intermediate value.

When called Alligation Medial; when Alternate.

300. When the quantities and prices are known, the employed process is called *Alligation Medial;* but when the proportional quantities of the mixtures are not known, except through their mean rates, it is called *Alligation Alternate.*

EXAMPLE 1.—A wine merchant mixed together 3 different kinds of wine, as follows: 3 casks at $40 per cask; 3 at $50; and 3 at $60: what is the mean price per cask? *Ans.* $50.

OPERATION.

40	50	60
3	3	3

120+150+180=$450;
then 9:450::1:50

EXPLANATION.

To find the value.

Having multiplied the cost of each cask by 3, and found by addition, the united worth, we say, as the sum of all the quantities (9) is to their whole value (450), so is any part (1) of them to its mean price.

2. A grocer mixed together three different kinds of coffee, as follows: 100lb. of Rio, at 10 cents per lb.;

250lb. of Mocha, at 20 cents per lb.; and 300lb. of St. Domingo, at 8 cents per lb.: what is the mean price of the mixture?

3. A grocer mixed together three different kinds of sugar, as follows: 450lb. at 6 cents per lb.; 740 at 7 cents per lb.; and 500lb. at 8 cents per lb.: what is the value of one pound of the mixture?

4. A goldsmith mixed three different kinds of gold, in these proportions: 4oz. of 14 carats fine; 5oz. 16 carats; and 6oz. 18 carats: how many carats of fine gold are in one ounce of the mixture?

5. A grocer mixed 40 gallons of water with 150 gallons of wine, valued at $1 per gallon: what is the value of the mixture?

6. A grocer mixes together two different grades of molasses: the first is worth 28 cents per gallon, but the second 35. The mixture contains 1gal. of the first kind to 2gal. of the second: what is the mixture worth per gallon?

7. What would the following mixture of corn be worth a bushel: 20 bushels at 78 cents per bush.; 25 at 65 cents per bush.; and 15 at 45 cents per bush.?

ALLIGATION ALTERNATE.

Alligation Alternate defined.

301. *Alligation Alternate* is a process showing what quantity or quantities of known values must be mixed with a given quantity of another, to produce a mixture of a specified name.

EXAMPLE 1.—A grocer wishes to mix wines, which are 14, 16, 22 and 28 shillings a gallon, in such form that the mixture shall be worth 20 shillings a gallon.

How to find the worth of mixtures.

OPERATION.

20	14	8
	16	2
	22	4
	28	6

8 gals. at 14s.
2 " " 16s.
4 " " 22s.
6 " " 28s. } *Ans.*

EXPLANATION.

Here we write the prices of the different wines in order, in a vertical form; and each price that is less than that of the mixture we connect by a line with one or more of the prices that are greater than it. We then write the difference between the required price

and each of those that are less, by the side of the larger one, with which it is connected; also, the difference between each larger price and the price of the mixture by the side of the less, in its connection. The difference standing by the side of each price is the quantity required to form the mixture.

Note.—By calculating the mean prices as given above = 400, and dividing by the gallons=20, we find by the quotient 20 the proof of the correctness of the calculation. Proof.

2. A grocer wishes to mix 4 different kinds of wine, to obtain a mixture that shall be worth \$1 per gallon. The first kind is worth 75 cents per gal.; the second, 50 cents; the third, \$1.25; and the fourth \$2: how many gallons must he take of each kind?

3. A grocer wishes to mix four different sorts of tea, worth 5s., 6s., 8s. and 9s. per lb., so as to have a mixture of 87lbs. worth 7s. per lb.: how much must he take of each kind? *Ans.* 21¾lb.

4. A goldsmith has 4 different kinds of gold, 14, 16, 20 and 22 carats fine; he wishes to obtain from them a mixture 18 carats fine: how many ounces must he take of each?

5. A drover has sheep worth \$2, \$4, \$6, \$8 and \$10 each: how many from these must he take to form a flock of 80 worth \$11 each?

6. A grocer wishes to mix 3 different kinds of sugar, to obtain a mixture worth 8 cents per lb. The first kind is worth 7 cents per lb.; the second, 6 cents; and the third, 10 cents: how many of each must he take to form the mixture?

7 A grocer wishes to mix 4 different kinds of coffee, so as to have a mixture worth 12 cents per lb. The first is worth 8 cents per lb.; the second, 10 cents; the third and fourth 14 cents: what quantity must he take of each?

TARE OR ALLOWANCE.

302. *Tare* is a mercantile term to express an allowance made in the transfer of goods for known or presumed deficiencies. Tare defined.

An allowance at ports of entry.

303. An allowance is made at the port of entry on dutiable goods, because boxes, etc., containing imported articles, are not considered any part of the commodities subject to tariff charge.

Allowance by merchants.

304. Besides this allowance or tare, deductions are made by wholesale merchants on boxes, etc., in the interchanges of trade.

The allowance of draft.

305. When goods are not actually entitled to these allowances, a deduction, commercially known as *draft*, is made for waste. This is 9 lbs. on the ton, and proportionally for smaaller weight.

Allowance for liquors.

306. On liquor in casks there is usually the allowance of 5 per cent. for leakage. On liquor in bottles (particularly porter and other fermenting liquors), for breakage, there is an allowance of 10 per cent.

Gross weight; Net weight.

307. Gross weight is the whole weight of the goods before allowance is made; and net weight is what remains after allowances are deducted.

EXAMPLE 1.—What is the net weight of 10 boxes of candles, each weighing 50lbs., there being a tare of 4lbs. on each box?

OPERATION.

50	10
10	4
500lbs.	40lbs. tare.
40	
460 net.	

Note.—Examples in Tare are performed by multiplication, subtraction, and in some cases, by proportion.

2. At $56 per hhd., what will 4hhds. of sugar amount to, allowing 10lbs. per cwt. tare on the gross weight of 19cwt., 3qrs.. 15lbs.?

3. At 16 cents per lb., what is the worth of 4 bags of coffee, the gross weight of which is 650lbs., allowing 2 lbs. tare on 100lbs.?

4. At 8 cents per lb., what is the cost of 3hhds. of sugar, weighing gross, 1cwt., 3qrs. 14lbs.; 2cwt., 2qrs. 13lbs.; 3cwt., 1qr. 15lbs., allowing tare of 9lbs. in each cwt.?

5. At $15 per cwt., what will 10cwt., 3qrs. 12lbs. sugar cost, allowing tare 10lbs. per cwt.?

Note.—Deduct tare from gross weight, and find answer by proportion: if 100lbs. cost $15, what will the number of lbs. net weight cost?

SINGLE POSITION.

308. *Single Position* is a process to ascertain the true answer to a question, by assuming a certain number as the rightful one, or as leading to it. **Single Position defined.**

EXAMPLE 1.—I have a certain number of sheep in my pasture, and if the number were increased by $\frac{1}{2}$ and $\frac{1}{3}$, the whole would be 66: what is the number? *Ans.* 36.

OPERATION.

Suppose 12 is the number, then by adding $\frac{1}{2}$ of $12=6$, and $\frac{1}{3}$ of $12=4$ to $12=22$, we find the supposed number was the true one by the proportion, $22 : 66 :: 12 : 36$.

REMARK.—The operation with 12, which gives as above, 22, enables us to obtain the evidently true answer in the fourth term of the proportion.

309. The examples in single position can be analyzed without trouble. Thus, in the above example, the number to be found is fractionally $\frac{6}{6}=1$: $\frac{1}{2}$ of $\frac{6}{6}=\frac{3}{6}$ and $\frac{1}{3}=\frac{2}{6}$, which added$=\frac{11}{6}=66$. Hence, if $\frac{11}{6}=66$, $\frac{1}{11}=6$, or $\frac{1}{6}$ of the number; then, as $\frac{6}{6}=$the whole, $6\times6=36$. *Ans.*

2. The master of a school being asked how many pupils he had charge of, said that if he had as many more as his present number, $\frac{1}{2}$ as many more, $\frac{1}{3}$ and $\frac{1}{2}$ as many more, he should have 296: what was the number? *Ans.* 96.

3. A man, who was asked his age, said that if $\frac{2}{5}$ of it were multiplied by 7, and $\frac{3}{5}$ subtracted from the product, the remainder would be 66: what was his age?

4. What is the number, which multiplied by 7, and the product divided by 6, will have a quotient of 14? *Ans.* 12.

5. Two men have the same income. One saves $\frac{1}{3}$ of his, but the other, by spending twice as much, finds himself, at the end of 4 years, \$560 in debt: what is the annual income? *Ans.* \$420.

6. Three speculators gained \$2400, of which B took 3 times as much as A, and C 4 times as much as B: what was the share of each?

DOUBLE POSITION

Double Position defined.

310. *Double Position* is a method to determine an answer by the use of two numbers, supposed to be the ones sought.

Results differ from those of Simple Position.

311. In this, the results vary from Single Position by not being proportional to the assumed numbers.

EXAMPLE 1.—A person having a certain sum, spent \$100 more than $\frac{1}{5}$ of it, and had remaining \$40 more than $\frac{1}{2}$ of it: what had he at first?

OPERATION.

Suppose, first, he had \$1500, then
\$100 more than $\frac{1}{5}$= 400=sum spent,

and 1100=remainder;
but \$40 more than $\frac{1}{2}$= 795

hence, \$305=1st error.

Suppose, second, he had \$2000, then
\$100 more than $\frac{1}{5}$= 500=sum spent,

and 1500=remainder;
but \$40 more than $\frac{1}{2}$= 1040

hence, \$460=2d error.

\$1500×460=\$690000=1st assumed No.×2d error.
\$2000×305=\$610000=2d assumed No.×1st error.

155) \$80000(\$516$\frac{20}{155}$. *Ans.*
775
250
155
950
930
20

that is, the difference of the products divided by the difference of the errors, gives the result of \$516$\frac{20}{155}$.

EXPLANATION.

By the example it will be noticed, that 1st, having supposed two numbers, we proceed with them according to the question; 2d, that having compared each result with that in the question, and calling each difference an error, we multiplied the 1st assumed number by the 2d error, and the 2d assumed number by the first error; and 3d, that we divided the difference of the products by the difference of the errors for the true answer.

Process explained.

A guide to similar examples.

REMARK.—Had one assumed number been too great and the other too small, we should have divided the sum of the products by the sum of the errors.

When the assumed numbers are too large or too small.

2. Three persons speaking of their ages, B said that he was 10 years older than A, and C, that his doubled both of theirs: what were their respective ages, the united sum being 100? *Ans.* A's 20, B's 30, C's 50.

3. What number is that, which being divided by 7, and the quotient diminished by 10, three times the remainder is 24? *Ans.* 123.

4. Two clerks have the same income; A saves ¼ of his yearly, but B, by improvidently spending $150 per annum more than A, at the end of 8 years finds himself $400 in debt: what are their incomes, and what the annual expenditures of each?

Note.—First, assume that each had $200; second, $300; then the errors will be 400 and 200.

Ans. $400 in.; A's exp. $300; B's $450.

5. A planter purchased a number of horses, mules and cows for $2640. He paid for each horse $50; for each mule $\frac{8}{5}$ as much as for a horse; and for each cow $\frac{1}{2}$ of the price of a horse. There were 3 times as many mules as horses, and twice as many cows as mules: what was the number of each?

MISCELLANEOUS EXAMPLES.

312. EXAMPLE 1.—A owes B $400, due in 3mo $250 in 1mo.: what is the mean time of payment?

2. What is the bank discount of $455.60, payable in 6mo., at the rate of 7 per cent.?

3. What commission is a factor to receive on the sales of 350hhd. of sugar, at $52 per hhd., at the rate of $2\frac{1}{2}$ per cent., and $2\frac{1}{2}$ per cent. for the guarantee of sale?

4. A, B and C jointly purchased a piece of land A paid $\frac{1}{2}$ of the price, B $\frac{1}{3}$, and C the remaining $\frac{1}{6}$ They subsequently determined to dispose of it, and gained $750: what was the gain of each?

5. A grocer wishes to mix three grades of sugar to obtain a mixture worth 8 cents per lb. The first is worth 6, the second 7, and the third 10 cents: what must he take of each to obtain the desired kind?

6. What is the amount of $375, at 7 per cent. interest, for 1yr. 6m.?

7 What is the amount of $400, at 7 per cent. compound interest, for 4 years?

8. A trader purchased 400 barrels flour, at $4.50 per bbl., payable in 6m. In order to gain 10 per cent. and give a credit of 8m., reckoning bank discount at 7 per cent. per an., what must he ask per bbl.?

9. There is a fish whose head weighs 14lbs., his tail weighs as much as his head and $\frac{3}{10}$ as much as his body, and his body weighs as much as his head and tail: what is its weight? *Ans.* 80lbs.

10. A, B and C form a partnership. A puts in $500 for 3m, B $600 for $2\frac{1}{2}$m., and C $300 for 5m. They gain $550: what is the share of each?

11. A merchant owes $600, due in 7m.; but, as he pays $\frac{1}{2}$ cash, and will pay $\frac{1}{3}$ in 4m., how long may he retain the balance? *Ans.* 2yr. 10m.

12. A person buys a farm for $2000, payable in 16 equal annual instalments, commencing the first year after the purchase. At the rate of 6 per cent., what sum paid down would fulfil his engagement?

Ans. $1263.23.

13. At 12 per cent. advance, what will $750 be in £ s. d.?

14. A watch sold for $60 lost 12 per cent. by the sale: what was the cost?

15. At £33 per cwt., what is the value of 6¼lbs.?

16. What is the interest of $2650, at 7 per cent., for 90 days?

17. What is the government tonnage of a single decker whose length is 50 feet, breadth 12½ feet and depth 10 feet?

18. What are the contents, in gallons, of a cask whose length is 44 inches, head diameter 28 inches, and bung diameter 36 inches?

19. A owes B $800, payable July 4, 1863, and B owes A $600, payable October 6, 1863: when is the equated time for settlement, and what should A pay at that date?

RADICAL ARITHMETIC.

PART SIXTH.

INVOLUTION.

Involution defined.

313. *Involution* is the process used to raise a number to some required power; thus, $2\times2=4$, the 2d power or square of 2; $2\times2\times2=8$, the 3d power or cube of 2.

What the power is.

314. The power is the product of a number multiplied by itself.

The index or exponent.

315. Sometimes the power is indicated by means of a small figure placed at the right hand of the root, slightly elevated, called the index or exponent; thus, 3×3 is written 3^2, and is read the square or 2d power of 3.

The root or basis number: 1st power; square; cube.

316. The number at the basis of the operation is known as the root or the 1st power; when once multiplied, it is known as the 2d power or square; when twice multiplied, as the 3d power or cube, and so on.

What the square and cube consist of.

317. The square of a number consists of twice as many figures as the root, or of one less than twice as many; the cube of a number of three times as many as the root, or of one or two less than three times as many.

Vinculum; Parenthesis.

318. A short horizontal line over two or more figures is called a vinculum; and, like a parenthesis, indicates that the numbers so connected are subject to a similar operation; thus, $\overline{8+2}\times3$, is the same as $(8+2)\times3=30$.

319. A number already raised to a power is involved by multiplying its index by the index of the power to which it is to be raised. The index multiplied by an index.

320. A vulgar fraction is involved by involving the numerator and denominator separately; thus, the 3d power of $\frac{4}{6}$ is $(\frac{4}{6})^3=(\frac{2}{3})^3=\frac{2^3}{3^3}=\frac{8}{27}$ How a vulgar fraction is involved.

321. The number of decimal places in the power of a decimal fraction equals the number of decimal places in the root multiplied by the index of the power; thus, the 3d power of .12 will contain 6 decimal places, for $.12\times.12=.0144\times12=.001728$, or $12^3=.001728$. What the number of decimal places equals

322. To divide a power of any number by any other power of the same number, we subtract the index of the divisor from that of the dividend; thus, $57\div5^3=5^4$ To divide powers.

EXAMPLE 1.—What is the 3d power of 4?

Ans. 64.

OPERATION.

$4\times4\times4=64$, or $4\times4=16\times4=64$.

2. What is the 6th power of 5?
3. What is the 4th power of 3? *Ans.* 81.

   $3\times3\times3\times3=81$, or $3\times3=9\times3=27\times3=81$.
4. What is the square of 12?
5. How much is the square of 10?
6. How much is 10 square?
7. How much is 10^2?
8. What is the product of $6^4\times6^2$?
9. What is the third power of $6^{2\,1}$? *Ans.* $\frac{8}{343}$.
10. What is the square of $\frac{1}{2}$?
11. What is the square of $5\frac{1}{2}$? *Ans.* $30\frac{1}{4}$.

    $5\frac{1}{2}=\frac{11}{2}\times\frac{11}{2}=\frac{121}{4}=30\frac{1}{4}$.
12. What is the value of 10^4?
13. What is the cube of 3? *Ans.* 27.

    $3\times3\times3=27$.
14. What is the cube of 4?
15. What is the cube of 25?
16. What is the square of $16\frac{1}{2}$?
17. What is the square of .25? *Ans.* .0625.
18. How many figures are in the cube of 99?

    Ans. 6.
19. How many figures are in the cube of 243?
20. How many figures are in the 5th power of 99?

323. A TABLE OF POWERS.

1st,	1	2	3	4	5	6	7	8	9
2d,	1	4	9	16	25	36	49	64	81
3d,	1	8	27	64	125	216	343	512	729
4th,	1	16	81	256	625	1296	2401	4096	6561
5th,	1	32	243	1024	3125	7776	16807	32768	59049
6th,	1	64	729	4096	15625	46656	117649	262144	531441
7th,	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th,	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th,	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th,	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401

EVOLUTION.

Evolution defined.

324. *Evolution*—a process the inverse of Involution—is used to find the root from the given power, and may be defined the Extraction of Roots.

How the root is indicated.

325. The root is indicated by the employment of what is called the fractional index, or by this symbol, which is known as the radical sign.

The index of the root.

326. A figure placed above this sign is the index of the root, and is the same as the denominator of the fractional index. When a number is not given in connection with the radical sign, 2 is to be understood.

A power and a root indicated together.

327. A power and a root can be indicated at the same time by the index and radical sign; thus, $\sqrt[3]{8^5}=32$, and is the cube root of the 5th power of 8.

Imperfect powers; surd numbers.

328. Some numbers cannot be extracted. Such are called imperfect powers, and their roots irrational, radical or surd numbers. For convenience, the term radical is employed, as before stated.

Perfect powers; rational roots.

329. Those are known as perfect powers that can be extracted, and their roots are called rational.

The first ten numbers and their squares.

330. The first ten numbers and their squares are,

1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

Explanation of above numbers.

331. The numbers in the first line are the square roots of those in the second; the numbers in the line second line are called perfect squares.

EXAMPLE 1.—What is the square root of 1024? *Ans.* 32. Process explained.

OPERATION.

$$
\begin{array}{r}
\dot{1}0\,\dot{2}4(32 \\
9 \\
\hline
62)1\,24 \\
1\,24 \\
\hline
\end{array}
$$

EXPLANATION.

We first point off the number into periods of two figures each, as we wish to find the squares of the tens and hundreds. We then find the greatest square in the 10, which=3 tens or 30. Squaring the 3, which gives 9 hundred, we place the 9 beneath the hundreds and subtract; this takes away the square of the tens and leaves 124. Next, we double the divisor, which is the root already found, and divide this remainder—without the right hand figure—by it, and have in the quotient, the units-figure of the root. Annexing this figure to the increased divisor, we multiply again, and find the desired result.

Note.—A similar course is to be pursued should there be more figures.

To extract the square root of a number is simply to resolve it into two equal factors; that is, to find a number which, multiplied into itself, will produce the given number. A simple expression of evolution.

2. How large a square floor can be laid with 576 square feet of boards?

OPERATION.

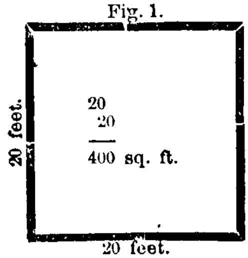
$$
\begin{array}{r}
\dot{5}7\dot{6}(24 \\
4 \\
\hline
44)176 \\
176
\end{array}
$$

EXPLANATION.

Here, the area being given, we are to find the length of one side.

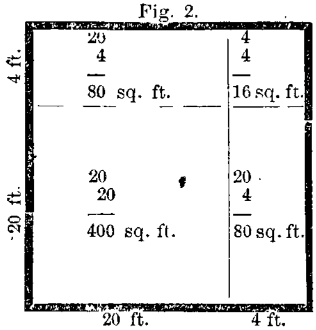
REMARK.—To make the operation intelligible, we form a square whose sides shall be 2 tens=20 feet in length. The area of this square, 20×20 =400 square feet, and this number taken from 576 leaves 176 to be employed in the required enlargement. This is made on the 2 sides, as in figure 2d (though it could be made on the four), and their breadth is alike, being twice

Fig. 1.



the tens of the root=4 or 40 feet. Now, 176 divided by 40 gives 4 feet, which, added to the trial divisor, 40=44, and is the entire length of the two sides, and 44×4=176; that is, the length of the addition multiplied by its breadth gives its area. Squaring the sides, each being 24, shows 576, and proves the sum.

Fig. 2.



When the product of divisor is in excess.

When the product of the divisor by a number is in excess of the dividend, make the quotient figure smaller.

When the number is not a perfect square.

When the given number is not a perfect square annex ciphers for new periods.

When the trial divisor exceeds dividend.

The quotient figure will be a cipher when the trial divisor is greater than its dividend.

3. What is the square root of 3606?

4. What is the square root of 390625?

5. What is the square root of 5764801?

6. What is the length of one side of a lawn which contains $3\frac{1}{2}$ acres, or 400 rods, if made into a square?

7 What is the square root of 15625?

8. If a square field contains 6400 square rods, what is the measurement on each side? *Ans.* 80.

9. What is the square root of 156.7325?

A number both integral and decimal.

REMARK.—A number, partly integral and partly decimal, is extracted in the same way as a whole number; the first point, in such case, must be placed over the units and extend both right and left; thus, 156.7325.

The number of integral figures in the root.

The number of integral figures in the root is as many as there are periods of integral figures in the power; and for each period of decimals in the power, there will be a decimal figure in the root.

10. What is the square root of 3271.4207?

Ans. 57.19.

11 What is the square root of .25?

12. What is the square root of 1.44? *Ans.* 1.2.

13. What is the square root of $\frac{1}{4}$? *Ans.* $\frac{1}{2}$.

REMARK.—We reduce a vulgar fraction to its simplest form, and then take the root of the numerator and denominator separately. To find the root of a vulgar fraction.

14. What is the square root of $30\frac{1}{4}$?

REMARK.—When either term, after being reduced, is an imperfect square, we change the fraction to a decimal, and proceed by directions already given. When either term is an imperfect square.

15. What is the square root of $\frac{3}{4}$? *Ans.* .866.
16. What is the square root of $\frac{108}{147}$? *Ans.* $\frac{6}{7}$.
17 What is the square root of $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}-\frac{9}{16}$?

$\sqrt{\frac{1}{2}+\frac{3}{4}+\frac{7}{8}-\frac{9}{16}}=\sqrt{\frac{25}{16}}=\frac{5}{4}=1\frac{1}{4}$ *Ans.*

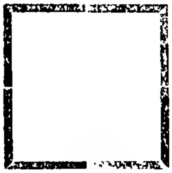
18. What is the difference between $\sqrt{9}$ and 9^2?
19. What is the difference between $\sqrt{16}$ and $\sqrt{9}$?
20. What is the difference between $\sqrt{\frac{1}{81}}$ and $\frac{1}{3}^2$?
21. What is the difference between $\sqrt{4}$ and $\sqrt{9}$? *Ans.* 5.
22. What is the sum of $\sqrt{30\frac{1}{4}}$ and $272\frac{1}{4}$?
23. What is the square root of $\sqrt{930\frac{1}{4}}$?

$\sqrt{930\frac{1}{4}}=\sqrt{\frac{3721}{4}}=\frac{61}{2}=30\frac{1}{2}$. *Ans.*

24. What is the difference between $\sqrt{81}$ and 81^2?

APPLICATIONS IN SQUARE ROOT.

DEFINITIONS.



332. A square is a figure with four equal sides and four equal angles, the angles being where the sides meet. Definition of a square.

The vertex.

333. The point of meeting is the vertex.

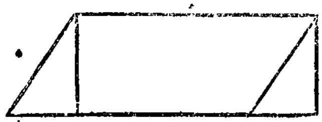
A right angle.

334. The sides or lines of a square being perpendicular to each other, make each angle a right angle.

EXAMPLE 1.—If an acre of land be laid out in a square form, what will be the length of each side in rods? $1 \times 4 = 4 \times 40 = 160$. *Ans.*

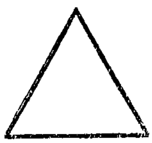
A parallelogram.

A parallelogram is a figure which has its opposite sides of equal length, and its opposite angles equal. In the figure, two parallelograms are described.



2. In a room 16 feet long and 11 feet wide, how many square feet are there? *Ans.* 176 feet.

A triangle.



A triangle is a plain figure of three sides and three angles.

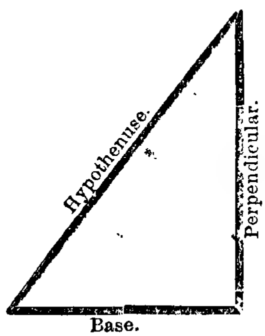
To find area of an irregular field.

The area of an irregular field is found by its being marked into triangles.

3. What is the area of a triangular piece of land, one side of which is 40 rods, and the distance from the corner opposite that side to the other, 20 rods? *Ans.* $\frac{20}{2} \times 40 = 400$ rods.

Definitions of a right angled triangle and its parts.

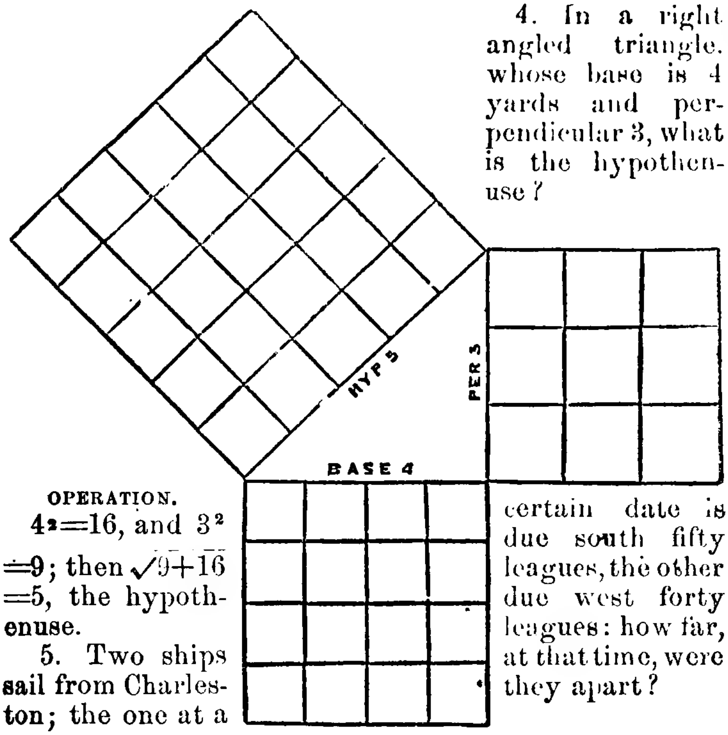
A right angled triangle has one of its angles a right angle. The side opposite the right angle is the hypothenuse, the lower line the base, the other the perpendicular.



What the square of the hypothenuse equals.

In a right angled triangle, the square of the hypothenuse is equal to the sum of the squares of the other two sides. This is practically apparent by the following diagram, in which the small squares on the hypothenuse, $25 =$ those of the base, $16 +$ those of the perpendicular, $9 = 25$.

4. In a right angled triangle, whose base is 4 yards and perpendicular 3, what is the hypothenuse? Diagram and operation describing the square of hypothenuse.



OPERATION.

$4^2=16$, and $3^2=9$; then $\sqrt{9+16}=5$, the hypothenuse.

5. Two ships sail from Charleston; the one at a certain date is due south fifty leagues, the other due west forty leagues: how far, at that time, were they apart?

Note.—Find the hypothenuse.

REMARK.—When the base and perpendicular are known, we find the hypothenuse by squaring the base and perpendicular, adding their results, and extracting the square of the sum. To find hypothenuse.

6. There is a street, in the middle of which, if a ladder 50 feet long be placed, it will reach a window 36 feet from the ground, on either side of the street: how wide is the street?

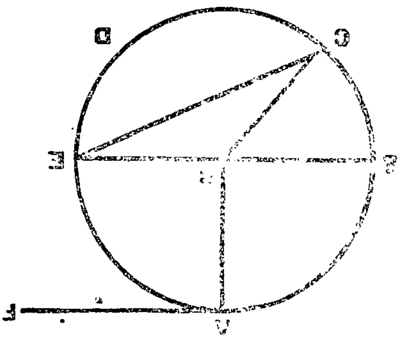
REMARK.—Having squared the hypothenuse and the known side, the square root of the difference will be the other side. When the hypothenuse and one side are known.

7 What is the height of a house which is reached by a ladder 50 feet long, standing in a street [illegible] feet wide?

8. If a line 125 feet long will reach from the top of a tower, 100 feet high, to the opposite side of the street, what is the width of the street?

9. What is the height of a pine, the line of the hypothenuse being 100 feet, and from the foot of hypothenuse to the base of the tree 57 feet?

A circle: **335.** A circle is a plain figure whose centre is everywhere equally distant from the bounding curve, called the circumference.



Its circumference. The divisions of a circumference. **336.** The circumference is divided into 360 equal parts, called degrees, and designated by a small cipher slightly elevated at the right of the number; thus, 360°. The semi-circumference is this equally divided, or 180°; the quadrant one fourth, or 90°; the sextant one sixth, or 60°; the octant one eighth, or 45°, and so on.

An arc. **337.** Any portion of the circumference, say C D E, is an arc.

A chord. **338.** The straight line, C E, connecting the extremities of an arc, is a chord.

A diameter. **339.** The line which passes through the centre of a circle is a diameter, as B E.

A radius. **340.** A straight line from the centre to the circumference is a radius, as G A, G C, G E

A tangent. **341.** A straight line, as A F, which touches the circumference in one point, A, and cannot touch it elsewhere, is a tangent.

EXAMPLE 1.—What is the area of a circle whose diameter is 6 feet and circumference 19 feet?

Ans. 28½ feet.

To find area of a circle.

OPERATION.

½ Diameter $=3$
½ Circumference $=9\frac{1}{2}$
27
1½
Or 3.141592   28½
9
28.274328=28½.

EXPLANATION.

We here, to find the area, multiply ½ the diameter by ½ the circumference, or we square ½ the diameter and multiply it by 3.141592dec.

2. What is the area of a circle whose diameter is 20 feet? *Ans.* 314.15.

3. What is the area of a circle whose diameter is 38 feet?

4. What is the area of a circle whose circumference is 314.1592?

314.1592÷3.141592=100, the diameter. .785398 (which is ¼ 3.141592)×100²=7853.98, the area. *Ans.*

5. The diameter of a circle is 25: what is the side of the inscribed square?

$\sqrt{\frac{25^2}{2}}=\sqrt{312.5}=17.677$. *Ans.*

6. The diameter of a circle is 36: what is the side of the inscribed square?

7. The circumference of a circle is 314.1592: what is the side of the inscribed square? *Ans.* 70.71.

CUBE ROOT

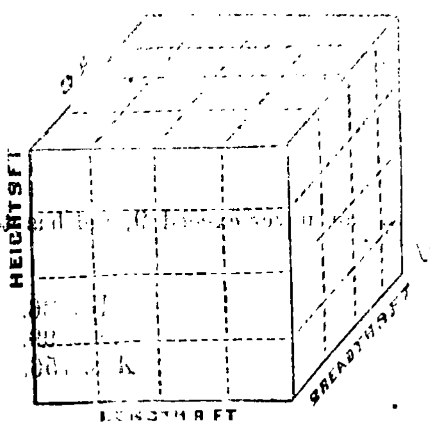
342. A *Cube* is a solidity of six equal sides, and each of these is an exact square. A cube.

343. The cube root of a number is that one which, multiplied into itself three times, produces the number whose cube is to be evolved. A cube root.

344. The numbers in the first line of the following order are the cube roots of the corresponding ones in the second. The last are perfect cubes. Cube roots and perfect cubes.

1, 2, 3, 4, 5, 6, 7, 8, 9.
1, 8, 27, 64, 125, 216, 343, 512, 729. Their numerical expression.

EXAMPLE 1.—What is the length of each side of a cubical block containing 729 cubic inches? *Ans.* 9.



OPERATION.

9×9×9=729; or, 9×81=729; hence, 9 is the number.

Note.—To find a cube is to find a number which, multiplied into itself three times, or multiplied once into its square, produces the given number. To find a cube

2. What is the length of each side of a cubical block, containing 1000 cubic inches? *Ans.* 10.

3. What is the cube root of 21024576? *Ans.* 276.

OPERATION.

To extract the cube root.

	$\dot{2}1\dot{0}24\dot{5}7\dot{6}$
1st trial divisor $=20^2\times3=1200$	8
$20\times7\times3=$ 420	
$7^2=$ 49	
1669	13024 1st dividend.
2d trial divisor $=270^2\times3=218700$	11683
$270\times6\times3=$ 4860	
$6^2=$ 36	
2d true divisor $=223596$	1341576 2d dividend.
	1341576

EXPLANATION.

First, We separate into periods of 3 figures each, the number, placing a point over units, thousands, etc.

Second, We find by trial the greatest cube in the left hand period, and placing its root as in square root, subtract the cube (8) from the left hand period, and to the remainder annex the next period for a dividend.

Third, We square the root figure, and, annexing two ciphers, multiply this result by 3 for a trial divisor; then we divide the dividend by the trial divisor, and set the quotient as the second figure of the root.

Fourth, We multiply the second root figure by the first, annex one cipher, and multiply this result by 3; then, adding the last product and the square of the last root figure to the trial divisor, we have in the sum the true divisor.

Fifth, We multiply the true divisor by the last root figure, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.

Sixth, We find a new trial divisor and proceed as before until all the periods have been used.

The true figure never above 9.

Note.—The true figure can never exceed 9, and has to be invariably found by trial.

4. What is the cube root of 17576? *Ans.* 26.

5. What is the cube root of 970299? *Ans.* 99.

6. What is the cube root of 3796416? *Ans.* 156.

7. What is the cube root of $\sqrt[3]{84.604519}$? *Ans.* 4.39.
8. What is the cube root of 74.088?

To extract the cube root of a vulgar fraction.

Remark.—To extract the cube root of a vulgar fraction, let the fraction first be reduced to its lowest terms, and then the cube root of the numerator and denominator be extracted separately, if exact; if they are not, let them be reduced to a decimal and then take their root.

9. What is the cube root of $\frac{54}{250}$?

$\frac{54}{250} \div 2 = \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$. *Ans.*

10. What is the cube root of $\frac{250}{686}$? *Ans.* $\frac{5}{7}$.
11. What is the cube root of $31\frac{15}{343}$? *Ans.* $3\frac{1}{7}$.
12. What is the cube root of $\frac{1}{5}$?

$3\sqrt{\frac{1}{5}} = \sqrt[3]{.2} = 58$. *Ans.*

13. What is the cube root of $\frac{27}{64}$?
14. What is the cube root of $\frac{1}{400}$?
15. What is the cube root of $\frac{1}{25}$?
16. What is the cube root of $3\frac{3}{8}$?

$\sqrt[3]{3\frac{3}{8}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2} = 1\frac{1}{2}$. *Ans.*

17. What is the cube root of $\frac{64}{74088}$?

345. Example 1.—What must be the length, depth and breadth of a box when these dimensions are alike, and the box contains 4913 cubic feet? *Ans.* 17

2. A jeweller has two small golden balls; one is 1 inch in diameter and the other two inches: how many of the smaller will it take to make the larger one? *Ans.* 8.

3. If a globe of gold, 1 inch in diameter, is worth $100, what is the diameter of a globe worth $7200? *Ans.* 8.

4. If the diameter of the sun is 886,144 miles and that of the earth 7912 miles, how many bodies like the earth will make one as large as the sun? *Ans.* 1,404,928.

5. If 1000 bodies like the earth are required to make 1 like the planet Saturn, and if the diameter of that planet is 79,000 miles, what is the diameter of the earth?

Ans. 7900 miles.

6. What is the length of one side of a cubical corn bin that contains 2500 bushels? *Ans.* 14.58 bushels.

7. What will be the length of one side of a cubical block which contains 1725 solid or cubic inches?

Ans. 12.

8. What will be the length of one side of a cubical block of granite, whose contents shall be equal to another 32 feet long, 16 feet wide, and 8 feet thick?

$\sqrt[3]{32 \times 16 \times 8} = 16$ feet. *Ans.*

MISCELLANEOUS EXAMPLES.

346. EXAMPLE 1.—What is the square root of 580644?
Ans. 762.

2. $\sqrt{1679616}$=how many? *Ans.* 1296.

3. How many figures are there in the cube of 99?
Ans. 6.

4. How many figures in the cube of 40?

5. What is the difference between $\sqrt[3]{8}$ and $\sqrt{4}$?

6. What is the difference between $\sqrt[3]{1}$ and 1^3?

7 A square table of mosaic contains 30,625 square stones of equal size: what number is in one of its sides?

8. A person wishes to form a tract of land, containing 50 acres, 2 roods and 20 rods, into a perfect square: what will be the length of each side?

9. A room is 25 feet long, 20 feet wide and 12 feet high: what number of square feet does it contain?

10. What is the cube root of $\frac{54}{250}$?

11. $\sqrt[3]{84.604519}$=how many? *Ans.* 4.39.

12. A pipe, $\frac{3}{4}$ of an inch in diameter, will fill a cistern in 5 hours: in what time will a pipe $2\frac{1}{4}$ inches in diameter fill it? *Ans.* $33\frac{1}{3}$min.

13. What is the cube root of 28?

14. A general with a force of 3136 men wishes to form them into a square: how many must he place in rank and file?

MISCELLANEOUS ARITHMETIC.

PART SIXTH.

ARITHMETICAL PROGRESSION.

347. *Arithmetical Progression* is the term that describes n increasing or decreasing series of numbers by the addi- ion or subtraction of the same figure. Arithmetical progression defined.

348. An increasing series, commencing with 2, is 2, 4, , 8, 10, 12, 14, etc. An increasing series.

349. A decreasing series, commencing with 14, is, by he subtraction of the same common difference, 14, 12, .0, 8, 6, 4, 2. A decreasing series.

350. The numbers used are the terms of the series or rogression. The first and last are the extremes, the thers the means. Terms: extremes; means.

351. In each Arithmetical Progression are five parts; hree of which being known, the two others can be found; hese are, Number of terms and names.

1st. The first term.
2d. The last term.
3d. The common difference.
4th. The number of terms.
5th. The sum of all the terms.

EXAMPLE 1.—A boy bought 20 marbles, and paid 1 sent for the first, 3 cents for the second, and so on: what number of cents did he invest in that purchase?

Ans. 39.

To find any designated term.

OPERATION.	EXPLANATION.
19 the number, less 1. 2 common difference. — 38 1 first term. — 39 last term.	Here, we multiply the common difference (2) by the number of terms preceding the required term; we then add the product to the first term (the series being an increasing one),

and find in the sum the answer sought.

When the series decreases.

Note.—When the series is a decreasing one, we vary from the above and subtract the product from the first term, and find answer in the difference.

2. If a man on a journey travel the first day $3\frac{1}{2}$ miles, the second 6, and so on in arithmetical progression, how far will he travel the 20th day? *Ans.* 51m.

3. A man had 5 sons whose ages had the same difference; the youngest was 6 years old and the oldest 26: what was the common difference of their ages? *Ans.* 4.

4. The extremes of an arithmetical series are 3 and 59, and the number of terms 8: what is the common difference? *Ans.* 8.

To find the common difference when extremes and number of terms are given.

OPERATION.	EXPLANATION.
$59-3=56\div 7=8$.	The extremes and number

of terms having been given, we found the common difference by the division of the difference of the extremes ($=56$) by the number of terms less 1 ($8-1=7$).

5. The extremes are 17 and 137, and the number of terms 9: what is the common difference?

6. The extremes are 24 and 180, and the number of terms 13: what is the common difference?

7. A man had 6 sons whose several ages differed alike; the youngest was 3 years old and the eldest 28: what was the difference of their ages?

8. A man has 10 sons whose ages form an arithmetical series; the youngest is 2 and the eldest 20: what is the difference of their ages?

9. If the extremes be 3 and 45, and the common difference 6, what is the number of terms? *Ans.* 8.

When the extremes and common difference are given.

OPERATION.	EXPLANATION.
$45-3=42\div 6=7+1=8$.	In this example, the ex-

tremes and the common difference being given, we divide the difference of the extremes ($45-3=42$) by the common difference (6)$+1=7$.

10. A man being asked the number of his children, id that the youngest was 8 and the eldest 36, and that e increase had been one in every 3 years: how many id he?

11. A stone falling descends $16\frac{1}{2}$ feet in the first second, d $209\frac{1}{12}$ in the last second; the increase of its velocity second being $32\frac{1}{6}$ feet: in how many seconds does it ll? *Ans.* 7

12. How many times does a clock strike in 12 hours? *Ans.* 78.

OPERATION.	EXPLANATION.
$1+12=13\times6=78.$	We multiply the sum of

e extremes $(1+12)$ by one half $(=6)$ the number of rms.

When the extremes and number of terms are given.

13. If a piece of land, 60 rods in length, be 20 rods de at one end, and at the other terminates angularly, iat is its number of square rods? *Ans.* 600.

14. The first term is 5, the common difference 8, and e number of terms 21: what is the sum of the series? *Ans.* 1685.

OPERATION.	EXPLANATION.
$8\times21=168$ Add 5 ——— 1685	We find, in this case, the last term by multiplying the common difference (8) by the number of terms

1) and adding first term to the product.

When the first term, common difference, and number of terms are given.

15. The first term is 7, the common difference 9, and e number of terms 25: what is the sum of the series?

16. The first term is $\frac{3}{4}$, the common difference $3\frac{1}{4}$, and e number of terms 65: what is the sum of the series?

17. A falling body descends $16\frac{1}{12}$ feet in the first second time, and the increase of velocity is $32\frac{1}{6}$ feet each suc-eding second: how far will it fall in 8 seconds?

18. The sum of an arithmetical series is 7, the number terms 8, and the least term 4: what is the greatest rm? *Ans.* 14.

OPERATION.	EXPLANATION.
$8\div2=4$; $72\div4=18$, and $18-4=14.$	We divide the

im of the series (72) by half the number of terms (4), id subtract the given extreme (4) for answer.

When the sum of a series, number of terms and one extreme are given.

19. The sum of a series is 264, the number of terms 2, and the greatest term is 39: what is the least term?

20. A falling body descends $1029\frac{1}{3}$ feet in 8 seconds; the 8th second it falls $241\frac{1}{4}$: how far does it fall in the st second? *Ans.* $16\frac{1}{12}$.

GEOMETRICAL PROGRESSION.

Geometrical progression defined.

352. *Geometrical Progression* is the term applied to that part of arithmetic which shows the increase, by multiplication, or decrease by division, of a numerical series through a common number.

The terms.

353. The numbers of these series are terms; the first and last are the extreme; the others mean.

The ratio; when an integer; when a fraction.

354. The common number employed is the ratio. It is an integer when the series increases, but a fraction when it decreases.

An increasing series; a decreasing series.

355. An increasing series, with the ratio 2 is, 1, 2, 4, 8, 16, 32, 64, 128; a decreasing series, with the ratio $\frac{1}{2}$, is, 128, 64, 32, 16, 8, 4, 2, 1.

Five terms and their names.

356. There are five terms in Geometrical Progression, any three of which known, easily determines the remaining two: these are,

1st. The first term.
2d. The last term.
3d. The number of terms.
4th. The sum of all the terms.
5th. The ratio.

Note.—By the ratio, we multiply or divide to form the series.

REMARK.—A proper understanding of this branch of numbers, requires the knowledge of Algebraic equations and logarithms. In this work, a few illustrative examples only are given.

EXAMPLE 1.—The first term is 3 and the ratio 2: what is the 6th term?

When the first term, ratio and number of terms are given, to find the last or any term.

OPERATION.

$$2\times2\times2\times2\times2=2^5=32$$
$$3 \text{ 1st term.}$$
$$96 \; Ans.$$

EXPLANATION.

We multiply the first term by that power of the ratio whose index is equal to the number of terms preceding the required term for answer.

REMARK.—It will be seen that the last term is equal to

the first term multiplied by the ratio, raised to a power less 1 than the number of terms.

2. The first term of a decreasing progression is 128, the ratio $\frac{1}{2}$, and the number of terms 7: what is the last term?

$(\frac{1}{2})^6=\frac{1}{64}$. $128\times\frac{1}{64}=\frac{128}{64}=2$. *Ans.*

3. A sum of money is to be divided among 10 persons; A is to have \$10, B \$30, and so on: what will the tenth receive? *Ans.* \$196,830.

4. A lad offered to purchase 17 oranges, and to pay for them at the rate of one cent for the first, 2 cents for the second, and so on in duplicate order: what was the cost of the 17th orange?

5. The extremes are 2 and 20,000, and the ratio 10: what is the sum of the series?

OPERATION.

20000—2=19998; 10—1=9; 19998÷9=2222; and 2222+20000=22222. *Ans.* When the extremes and ratio have been given.

EXPLANATION.

In this example we divide the difference of the extremes (19998) by the ratio less 1 (10—1=9), and add to the quotient the greater extreme.

6. A man trading for a horse offered to pay for him at the rate of a cent for the 1st nail in his shoes, 3 for the 2d, and so on; there were 32 nails: what did the horse cost? *Ans.* \$9,265,100,944,259.20.

7. A father, at the celebration of his daughter's birthday in January, gave her \$5, and said he would double it on the first day of each successive week in the fifth year: what was the sum that he, through ignorance of Geometrical Progression, pledged himself to pay?

PERMUTATION AND COMBINATION.

357. *Permutation* is a process to find the different ways in which numbers or things can be placed; *Combination*, their various arrangements in sets or series. Permutation and combination defined

EXAMPLE 1.—In how many different ways can we arrange the first five letters of the alphabet? *Ans.* 120.

To find permutations.

OPERATION.	EXPLANATION.
1. a b c d e 2. b c d e a 3. d e a b c 4. e a b c d, etc. $1\times2\times3\times4\times5=120$. a b c d e=5 letters.	We multiply together all the terms of the natural series from 1 up to the given number, and find, in the last product, the number of changes sought.

2. How many different integral numbers may be expressed by writing once in each number, the 9 digits in succession? *Ans.* $1\times2\times3\times4\times5\times6\times7\times8\times9=362880$.

3. The solar spectrum consists of 7 colors—red, orange, yellow, green, blue, indigo and violet: in what varieties can these be placed? *Ans.* 5040

4. How many changes can be rung on St. Michael's bells, supposing them to be 8, and allowing 3 seconds to each round?

5. How many different companies, each of 7 men, may be selected from 21 men?

OPERATION.

$$\frac{21\times20\times19\times18\times17\times16\times15}{1\times\ 2\times\ 3\times\ 4\times\ 5\times\ 6\times\ 7}=116280.\ \textit{Ans.}$$

EXPLANATION.

To find the number of combinations of any number of things.

Here we form a 1st series of numbers, commencing with that (21) to be selected from, and decreasing as many times as the other number expresses; and a 2d series, commencing with 1, increasing to the number to be combined for a divisor, and find in the quotient the combinations sought.

6. The graduating class of the University of the South consists of 80 members, 12 of whom are to have honors and appointments: how many different selections could be made?

7 There are said to be 56 different elements in nature: if one particle in each element will combine with one particle in each of the other elements, how many combinations may be formed? *Ans.* 1540.

MENSURATION.

358. *Mensuration* is that part of arithmetical science which describes capacities of various kinds. Mensuration defined.

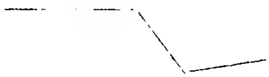
REMARK.—The points, lines and surfaces named in mensuration are imaginary, and are, simply, aids to a mathematical inquiry. Points, lines, etc., not real.

DEFINITIONS.

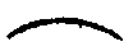
359. EXAMPLE 1.—A point has neither length, breadth nor thickness, but only position. A point.

2. A line has length, but no breadth or thickness. A line.

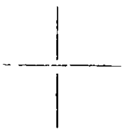
3. A right line, or straight line, extends only in one direction. A right or straight line.



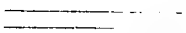
4. A broken line is formed of two or more right lines. A broken line.



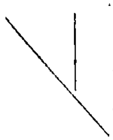
5. A curved line is one that constantly changes its direction. A curved line.



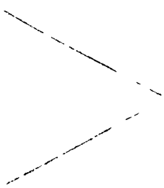
6. Two lines are perpendicular to each other, when they touch so as to form right angles. Perpendicular lines.



7. Two parallel lines are everywhere equally distant from each other. Parallel lines.



8. Two lines are oblique to each other when their point of union makes acute or obtuse angles. Oblique lines.



9. A surface, superficies, or area, has length and breadth, but no thickness, and is plane or curved. A surface, superficies, or area.

10. A plane surface is such, that if any two points are assumed upon it, the straight line joining the points will be wholly upon the face. A plane surface.

11. A curved surface constantly changes direction, as the exterior of a globular body. A curved surface.

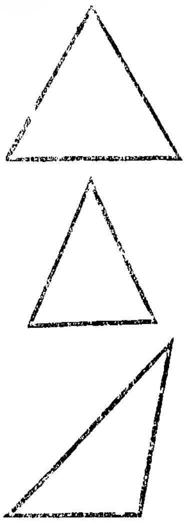
A polygon.

12. A polygon is a plane figure bounded by at least three straight lines.

Polygons of various sizes.

13. A polygon of 3 sides is called a triangle; of 4, a quadrangle; of 5, a pentagon; of 6, a hexagon; of 7, a heptagon; of 8, an octagon, etc.

Triangles, equilateral; isosceles; scalene.



14. Triangles with 3 equal sides, are equilateral; with 2, are isosceles; with 3 unequal sides, are scalene.

Note.—For measurement of triangles see 180th page.

To find the area of a parallelogram, rectangle or square.

360. EXAMPLE 1.—What is the area of a rectangular field, whose length is 40 rods and breadth 20?

Ans. 5a.

OPERATION.	EXPLANATION.
40 20 —— 160)800(5 acres. 800 ——	We multiply the base by the altitude, and have the answer, 800 rods. This reduced by the

number of square rods (4×40) in a square acre, gives 5.

Note.—For definitions of square, etc., see Art. 332–334.

2. What is the area of a parallelogram whose base is 2 feet and height 3 inches? *Ans.* 72 sq. in.

3. What are the contents of a field 60 rods square?

4. The parallel sides of a trapezoid are 7 and 11 feet, and its height or altitude 4 feet: what is its area?

Ans. 36 sq. ft.

A trapezoid.

Note.—A trapezoid is a 4 sided figure, having two of its sides parallel.

To find the area of a trapezoid.

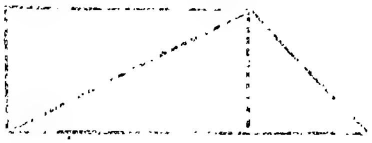
OPERATION.	EXPLANATION.
11+7=18×4=72÷2=36.	Here we multiply the sum

of the two parallel sides (=18) by the altitude, and divide the product by 2, which gives the area.

5 What is the area of a trapezoid whose parallel sides are 20.5 and 12.25, and its altitude 10.75yds.?

Ans. 176.031 sq. in.

REMARK.—The diagonal of a trapezoid divides it into two triangles whose bases are the parallel sides of the trapezoid, and whose common altitude is the altitude of the trapezoid. *The diagonal of a trapezoid.*



6. What is the area of a trapezoid whose altitude is 8, and whose parallel sides are 12 and 18? *Ans.* 120.

7 What is the area of a triangle whose base is 50 feet and altitude 8 feet? *Ans.* 200 sq. ft.

Note.—For definitions of a triangle see Art. 358, 14 Def.

OPERATION.	EXPLANATION.
$50\times4=200.$	We multiply the base by half $(8\div2=4)$ the altitude.

To find the area of a triangle.

8. What are the contents of a triangular field whose base is 25 rods and altitude 18 rods?

9. The sides of a triangle are 8, 10 and 12 feet: what is its area?

OPERATION.

$8+10+12=30\div2=15$; $15-8=7$
$15-10=5$; $15-12=3$; $\sqrt{15\times7\times5\times3}=\sqrt{1575}=40$ sq. ft. nearly.

EXPLANATION.

From the ½ sum of the 3 sides we subtract each separate side, and then, by multiplying together the ½ sum (=15) and the three remainders (7, 5, 3), we have in the square root of the continued product the area. *To find area when three sides of a triangle are known.*

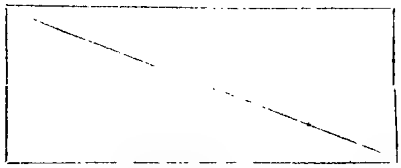
10. The sides of a triangle are 7, 12 and 15: what is its area? *Ans.* $\sqrt{1700}=41.23+$

11. The sides of a triangle are 10, 15, 20: what is its area?

12. What is the area of a parallelogram whose base is 30 feet and altitude 8 feet: and what, if diagonally marked, would be the areas each of the triangles formed by such line?

The diagonal of a parallelogram.

Remark.—The diagonal of a parallelogram separates it into 2 equal triangles, whose bases and altitudes are each equal to the base and altitude of the parallelogram.



To find area of a parallelogram; also, when triangled.

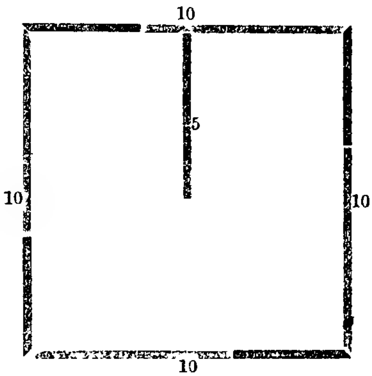
OPERATION.	EXPLANATION.
$30\times8=240$ $240\div2=120.$	The base, multiplied by altitude, gives the first answer; that, divided by 2 $(=\frac{1}{2})$, the second.

Note.—The area of a triangle is $\frac{1}{2}$ the area of a parallelogram that has the same base and altitude.

13. What is the area of a parallelogram whose base is 35 feet and altitude 10? what, if diagonally marked, would be the areas of its triangular divisions?

14. The sides of a square are 10 feet and the apothem 5 feet: what is its area? *Ans.* 100 sq. ft.

The apothem of a polygon.



Note.—The apothem of a polygon, or a figure with many angles, is the perpendicular drawn from the centre of the polygon to the middle of either side.

To find area of any regular polygon.

OPERATION.	EXPLANATION.
$10\times4=40\times2\frac{1}{2}(=\frac{1}{2}\ 5)=100.$	Here, the perimeter, or bounding lines of the figure, is multiplied by $\frac{1}{2}$ its apothem.

15. The sides of a square are 30 feet and the apothem 15 feet: what is its area?

The areas of all similar figures are to each other as the squares of their homologous or proportionate sides. By the following table of the areas of regular polygons, when each side is a unit, we can easily calculate the areas of figures therein named:

Name.	No. Sides	Apothem.	Area.
Triangle	3	0.2886751	0.4330127
Square	4	0.5000000	1.0000000
Pentagon	5	0.6881910	1.7204774
Hexagon	6	0.8660254	2.5980762
Heptagon	7	1.0382607	3.6339124
Octagon	8	1.2071068	4.8284271
Nonagon	9	1.3737387	6.1818242
Decagon	10	1.5388418	7.6942088
Undecagon	11	1.7028436	9.3656399
Dodecagon	12	1.8660254	11.1961524

A table for facilitating areal calculations of polygons.

16. The side of an octagon is 10 feet and its apothem 12.07106: what is its area? *Ans.* 482.8427 sq. in.

OPERATION.	EXPLANATION.
$10\times10=100\times4.8284271=482.842+$	We square

one side of the polygon whose area is required, and multiply the square by the tabular area of the polygon named, for area.

To find area of a polygon by the table.

17 What is the area of a regular triangle, one side being 8 inches?

18. What is the area of a square, one side being 12 inches?

19. What is the area of a heptagon, one side being 3 feet?

20. What is the area of an octagon, one of the sides measuring 8 rods?

21. What is the circumference of a circle whose diameter is 5 miles? *Ans.* 15.7080 miles.

To find the circumference of a circle.

OPERATION.	EXPLANATION.
$5\times3.1416=15.7080$.	The diameter is multiplied

by 3.1416dec., and gives very nearly the circumference.

Note.—For definitions of a circle and its divisions see Arts. 335–341.

22. What is the circumference of a circle whose diameter is 30 feet? *Ans.* 94.2480 feet.

Note.—The diameter of a circle is found by the inverse operation: $94.2480\div3.1416=30$.

To find the diameter of a circle.

23. What is the circumference of a circle whose diameter is 20 miles?

24. What is the area of a circle whose diameter is 7?

Ans. 38.4846.

To find the area of a circle when the diameter is given.

OPERATION.	EXPLANATION.
$7\times7=49\times.7854=38.4846$.	Multiply the square of the diameter by the decimal .7854.

25. What is the area of a circle whose diameter is 9?

Ans. 63.6174.

26. What is the area of a circle whose diameter is 20 miles?

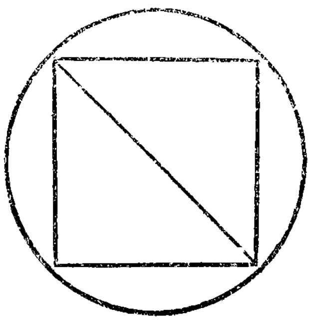
27 The diameter of a circle is 20 feet: what is the side of the inscribed square?

Ans. 14.142ft.

To find the side of a square inscribed in a circle.

OPERATION.

$20^2=400$; $400\div2=200$; and $\sqrt{200}=14.142$.



EXPLANATION.

We extract the square root of $\frac{1}{2}$ of the square of the diameter.

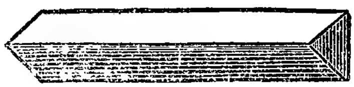
28. What is the side of the greatest square stick of timber that can be hewn from a cylindrical log 36 inches in diameter?

29. What is the convex surface of a prism whose base is bounded by 7 equal sides, each being 33 feet, and the altitude 22?

Ans. 5082 sq. ft.

A prism defined.

REMARK.—A prism is a solid* with two similar equal parallel faces, called bases, and its other faces parallelograms.



To find the convex surface of a right prism.

OPERATION.	EXPLANATION.
$33\times7=231\times22=5082$.	Here multiply the perimeter of base by the altitude.

When it is right; when oblique; when triangular, etc.

Note.—A prism is called right when its edges are perpendicular to its bases; when not perpendicular, oblique. It is also triangular, quadrangular, etc., according to its bases.

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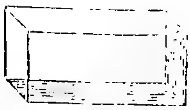
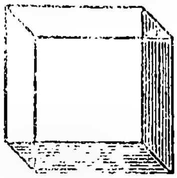
* A solid is a figure which has length, breadth and thickness.

30. What is the convex surface of a prism when there are 6 equal sides, each 18 inches in length, and in altitude 14 inches?



Note.—When a prism is bounded by 6 parallelograms, it is called a parallelopipedon.

A parallelopipedon.



When these 6 parallelograms are rectangles, the parallelopipedon is rectangular; when equal rectangles, it is a cube.

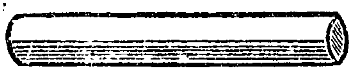
When it is rectangular; when a cube.

31. What is the convex surface of a prism with 6 equal sides, each 14 inches long, and the altitude 9 inches?

32. What is the convex surface of a cylinder whose altitude is 50 feet and the diameter of whose base is 20 feet? *Ans.* 3141.6 sq. ft.

Note.—A cylinder is a round body, whose diameter is unvariable, and whose ends are equal and parallel circles.

A cylinder defined.



OPERATION.

3.1416 decimal.
20 diameter.

62832
50 altitude.

3141.600

EXPLANATION.

Here, as we did to find the diameter of a circle, Art. 330, 22 Ex., we multiply 3.1416 by the diameter, and that result by the altitude.

To find convex surface of a cylinder.

33. What is the convex surface of a cylinder whose altitude is 1 foot, and the circumference of whose base is 1 foot and 6 inches? *Ans.* 216 sq. in.

34. What are the contents of a cylinder whose base diameter is 14 and altitude 25? *Ans.* 3848.4.

OPERATION.

$14 \times 14 = 196 \times .7854 = 153.9384 \times 25 = 3848.4$

EXPLANATION.

In this case, having found the area of the base by multiplying the square of 14 by decimal .7854, we find the contents by multiplying that result by the altitude.

To find the contents of a cylinder.

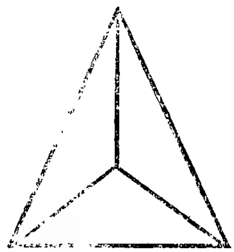
35. What are the contents of a cylinder whose base diameter is 18 and whose altitude is 8?

36. How many inches in a cylinder whose base diameter is 15 and whose altitude is 6?

37. What are the contents of a pyramid whose area base is 60 and altitude 18? *Ans.* 360.

A pyramid defined;

its vertex.



Note.—A pyramid is a solid having a polygonal face, called the base. Its other faces are called triangles, and meet at a common point, the vertex.

OPERATION.

$60 \times 18 = 1080 \div 3 = 360.$

EXPLANATION.

To find the contents of a pyramid.

The area base is multiplied by the altitude, and $\frac{1}{3}$ of that taken for the contents.

38. What are the contents of a pyramid, the area of whose base is 350 and the altitude 25?

39. What is the convex surface of a right pyramid whose slant height is 28 feet, and the circumference of whose base is 42 feet? *Ans.* 882 sq. ft.

OPERATION.	EXPLANATION.
$42 \times 21 = 882.$	The circumference of the

To find the convex surface of a right pyramid.

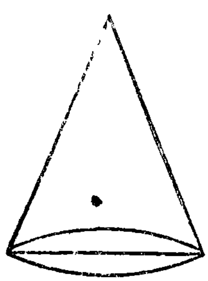
base is multiplied by $\frac{1}{2}$ of the slant height.

40. What is the convex surface of a right pyramid whose slant height is 56 feet, and the circumference of whose base is 60ft.?

41. What are the contents of a cone whose altitude is 30 feet and the diameter of whose base is 10ft.?

Ans. 785.40 sq. ft.

A cone.



Note.—A cone is a pyramid with circular base.

OPERATION.

$10^2 \times .7854 = 785400 \times 30 =$
$23562000 \div 3 = 785.40.$

EXPLANATION.

Here, the square of the base multiplied by decimal .7854, and that result by the altitude, with a division by 3, give the contents.

42. What are the contents of a cone whose altitude is 25 feet, and the diameter of whose base is 8 feet?

Ans. 418.88 sq. ft.

43. What is the convex surface of a right cone whose slant height is 90 feet and the circumference of whose base is 60 feet? *Ans.* 27 sq. ft.

OPERATION.	EXPLANATION.
$60 \times 45 = 2700.$	We multiply the circumference of the base by $\frac{1}{2}$ the slant height.

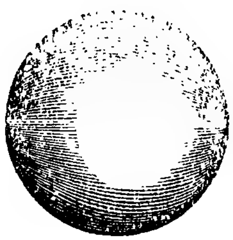
To find the convex surface of a right cone.

44. What is the convex surface of a right cone, whose slant height is 36 feet and base circumference 18 feet?

Note.—A cone is called right, when its edges are perpendicular to its bases; otherwise, it is oblique.

When a cone is right; when oblique.

45. What is the surface of a sphere whose diameter is 8? *Ans.* 201.06.



Note.—A sphere is a solid body bounded by a curved surface, all parts being equi-distant from a point within, called the centre. Its diameter is a straight line passing through its centre. All diameters of the same sphere are equal.

A sphere.

Its diameter.

REMARK.—The surface of a sphere equals 4 great circles of the same sphere; and, as we find the area of a circle by multiplying (Ex. 1, App. Sq. R.) the circumference by $\frac{1}{4}$ diameter, so we find a spherical surface, as shown in the explanation.

OPERATION.	EXPLANATION.
$8^2 = 64 \times 3.1416 = 201.06.$	In this case we square the diameter, and, multiplying it by 3.1416dec., find the surface.

To find the surface of a sphere.

46. What is the surface of a sphere whose diameter is 9ft.? *Ans.* 254.46 sq. ft.

47. If the sun's diameter is 896,000 miles, what is its surface? *Ans.* 2,522,120,323,072 sq. m.

48. What are the contents of a sphere whose diameter is 5ft.? *Ans.* 65.450 sq. ft.

OPERATION.	EXPLANATION.
$5^2 \times 3.1416 = 78.540 \times 5 = 392.700 \div 6 = 65.450.$	We multiply the surface by the diameter, and divide that by 6.

To find the contents of sphere.

49. What are the contents of a sphere whose diameter is 20 feet?

50. What are the contents of a sphere whose diameter is 28 rods?

51. What are the contents of a sphere whose diameter is 8000 miles?

MISCELLANEOUS EXAMPLES.

361. EXAMPLE 1.—A merchant bought 12 cases of merchandise for \$569: what would 25 have cost at the same rate?

2. If 8yds. of cloth cost £20 18s. 5d., what is that per yard?

3. If $9\frac{3}{5}$ barrels of flour cost £21 3s. 8d., what would $17\frac{5}{9}$ cost?

4. If $\frac{4}{7}$ of a ship is worth £865 5s. 9d., what is the whole worth?

5. What number, multiplied by $\frac{1}{2}$ of itself, will produce $7\frac{1}{2}$?

6. What number, multiplied by $\frac{1}{3}$ of itself, will produce $5\frac{1}{3}$?

7. A man had 4cwt. 3qrs. 15lbs. of tobacco, which, equally divided, he put into two parcels: what was each division?

8. Multiply $\frac{7}{13}$ of $7\frac{4}{11}$ by $\dfrac{5\frac{4}{7}}{2\frac{5}{11}}$. *Ans.* 9.

9. If, when wheat is 6s. 4d. per bushel, the penny loaf weighs 8oz., what ought it to weigh when wheat is 5s. per bushel?

10. How many gallons will a cistern contain, the diameter of whose base is 10 feet and the altitude 30?

11. If a staff 4 feet long cast a shadow of 6ft. 8in., what is the height of a column which casts a shadow of 155 feet at the same hour?

12. If 46 gallons of water run in an hour into a cistern containing 216 gallons, and 38 are drawn off in an hour, in what time will it be filled?

13. What is the square root of 9 times the square of 16?

14. A and B go from the same place and travel the same road; but A commences his journey 5 days before B, and travels at the rate of 29 miles a day; B follows at the rate of 45 miles a day: in how many days will B overtake A?

15. How many cubic feet in a cistern 4ft. 2in. long, 3ft. 8in. wide, and 5ft. 7in. high?

16. What is the square root of the square root of $\frac{9}{25}$ of the square of $\frac{12}{125}$? *Ans.* $\frac{6}{25}$.

17 A father, in his will, left \$15,000; of this, $\frac{1}{2}$ was for his eldest son, $\frac{1}{4}$ to the second son, and $\frac{1}{6}$ to his

daughter, and the rest in charities: how was it apportioned?

18. A man owes B \$500, to be paid as follows: \$150 in 8 months, \$225 in 6 months, and \$125 in 4 months: if paid at one time, what would the term for payment be?

19. A house is 50ft. from the ground to the eaves: what length of ladder would it take to reach the eaves, if the foot of the ladder cannot be brought nearer the house than 30 feet?

20. If a pipe 6in. in diameter will discharge a certain quantity of water in 4 hours, in what time will 3 four inch pipes discharge the double quantity? *Ans.* 6 hours.

21. A gentleman left an estate of \$17 500 between his widow and son; the son's share was $\frac{6}{9}$ of the widow's: what was the share of each?

22. What are the prime factors of 1400?

23. What is the hour when the time past from midnight is equal to $\frac{5}{11}$ of the time to noon?

24. What are all the integral factors of 1100?

25. How many square feet in a floor 12ft. 6in. wide and 14ft. 8in. long?

26. What is the greatest common measure of 1027 and 1781?

27 How much wood is there in a pile 4ft. wide, 2ft. 9in. high, and 16ft. 6in. long?

28. What is the duty on 300 bags of coffee, each weighing, gross, 160lbs., valued at 7 cents per lb., 2 per cent. being the tare allowed and 20 per cent. the duty?

29. What is the square of the cube root of $\frac{1}{5}$ of $\frac{3\frac{1}{4}}{5\frac{1}{2}}$ and $\frac{3}{2}$ of $\frac{2}{16\frac{4}{5}\frac{4}{2}\frac{8}{9}}$? *Ans.* $\frac{4}{9}$.

30. How many bricks 9in. long, $4\frac{1}{2}$in. wide and 2in. thick, will build a wall 6ft. high and $13\frac{1}{2}$in. thick, round a yard, each side of the same being 280 feet on the outside of the wall?

31. What is the interest of \$276 for 3 years, at 7 per cent.?

32. Two men speaking of their ages, one said that $\frac{2}{3}$ of his age equalled $\frac{2}{5}$ of the other's, and that the sum of both was 90: what were their ages?

33. An invoice of goods was sent from England with instructions to sell them and invest the proceeds in cotton, after deducting a commission of $1\frac{1}{2}$ per cent. on the sales, and 1 on the purchase of cotton. The goods were sold at an advance of 5 per cent. on the invoice price and the

amount of \$12,600 was received: what was the invoice price, and what sum was invested in cotton?

Ans. Invoice, \$12,600; Investment, \$12,288.11$\frac{82}{101}$.

34. Divide \$450 among 4 persons, so that when A has $\frac{1}{2}$ dollar B has $\frac{1}{3}$, C $\frac{1}{4}$ and D $\frac{1}{5}$.

35. Said A to B, my horse and saddle together are worth \$150; but my horse is worth 9 times as much as the saddle: what was the value of each?

36. A drover, with beeves and sheep, was asked how many he had of each kind. He said there were 174, but that the beeves were $\frac{9}{20}$ of the whole: how many of each was in the drove?

37 What is the cube of the square root of $\frac{2}{3}$ of $\frac{5\frac{3}{5}}{4\frac{2}{3}}-\frac{1}{3}$ of $\frac{7\frac{1}{2}}{450}$?

38. A grocer mixed 95lbs. of sugar that was worth 7 cents per lb., 65 that was worth 9 cents per lb., and 25 worth 12 cents per lb.: what was the mixture worth per lb.?

39. A man driving some sheep, cows and oxen, being asked the number of each kind, said that he had twice as many sheep as cows and three times as many cows as oxen, and that the whole was 80: what number of each kind did he have?

40. A colonel forming his regiment into a hollow square, found that, when he arranged the men 3 deep on each side of the square, he had 114 men left; but when he arranged them 4 deep, he wanted 114 men to complete the arrangement: what number did his regiment consist of? *Ans.* 750.

41. A merchant has due a certain sum of money, of which $\frac{1}{6}$ has to be paid in 2 months, $\frac{1}{3}$ in 3 months, and the rest in 6 months: in what time ought the whole to be paid?

42. A man built a house of 4 stories; in the lower story there were 16 windows, each containing 12 panes of glass, each pane 16in. long, 12in. wide; in the second and third stories there were 18 windows, each of the same size; in the fourth story there were 18 windows, each containing 6 panes, 18 by 12: how many square feet of glass were there in the house?

43. What is the difference between 3 times 6 and 18, and 3 times 18 and 6?

44. What number added to the 25th part of 2600 will make the sum 145?

45. A merchant has spices at 9d. per lb., at 1s., at 2s.,

at 3s. per lb.: how much of each kind must he mix to sell the same at 1s. 6d. per lb.?

46. What is the difference between 6 dozen dozens and ½ dozen dozens?

47. Write two millions, two hundred and two thousand, two hundred and twenty-two.

48. What is the value of $\frac{3}{9}$ of a cwt.?

49. What is the value of .325 of £1?

50. Reduce 12s. 3d. 2qrs. to the decimal of a pound.

51. What is the difference between $\sqrt{\frac{2}{18}}$ and $\frac{2^2}{18}$?

52. What is the value of $\sqrt{360\,000}$?

53. Three merchants, A, B and C, freight a ship with tobacco. A put on board 300 tons, B 150 and C 85. In a storm there was thrown overboard 125 tons: what was the loss to them severally?

54. How many boards 20 feet long and 15 inches wide will it take for a floor 40 feet long and 30 wide?

55. A grocer bought a hogshead of brandy for $87 on 6m. credit, and sold it for cash, with an advance on cost of $18: how much was his gain, allowing money to be worth 7 per cent. per an.?

56. What is the difference of time between May 10, 1860, and August 15, 1865?

57 How many hours from October 10, 1860, at 4 P M., to January 1, 1863, at 7 A. M.?

58. Two men hired a pasture for $35; A put in 3 horses for 4m., and B 5 horses for 3m.: what ought each to pay?

59. A grocer bought a hogshead of molasses for $30, but 8 gallons having leaked out, he wished to sell the remainder, so as to gain 4 per cent. on the whole cost: at what price per gallon must it be sold?

60. Divide $750 between 3 persons, so that the second shall have $\frac{4}{7}$ as much as the first, and the third $\frac{1}{2}$ as much as the other two.

61. A merchant sold a piece of cloth for $45, and lost by the sale 8 per cent.: what did the cloth cost?

62. A gentleman being asked the time, said that the time past noon was equal to $\frac{1}{5}$ of the time past midnight: what was the hour?

63. What is the discount of $260.85 for 1 year and 3 months, when interest is 8 per cent.?

64. A person was born on the 1st day December, 1814, at 4 o'clock in the morning: what was his age 1st day May, 1861, at 10 o'clock in the morning?

65. What is the area of a square piece of land, the sides of which are 27 chains? *Ans.* 72a. 3r. 24p.

66. A merchant shipped 14 bales of cotton at £30 10s sterling, 4¼hhd. of tobacco at £14 8s. 3d. per hhd., 210bbl. of rice at £3 9s. 6d. per bbl.: what was the amount in American money, with exchange 5 per cent.?

67. An inclined plane is 40 feet long and 5 feet high: what weight will be balanced by a power of 200 feet? *Ans.* 1600.

68. A lot of land, measuring 39 feet by 16½, cost $2500: what was that a square foot?

69. What is the area of a parallelogram, the height being 360 feet and width 271 yards? *Ans.* 32,520 sq. yd.

70. A merchant owes in England £350 15s. 3d., but has shipped 15 bags of cotton at the valuation of $90 each: what is the remaining indebtedness?

71. What is the area of a circle whose diameter is 10?

72. What is the commission on the sales of 250hhd. of sugar at $55 per hhd., at 3½ per cent., and 2½ per cent. for guarantee of sales?

73. There is due in England, for a bill of merchandise, £250 sterling: how many dollars must be remitted to pay for the bill, reckoning a £ at $4.87?

74. What is the area of a circle whose diameter is 14? *Ans.* 76,968.

75. There are due in Paris, for several pictures, 4565 francs: what amount in dollars and cents must be remitted, reckoning 19½ cents to the franc?

76. A note of $350, dated October 7th, and payable 6 months from date, is to be discounted on the 22d day October, at the bank discount of 7 per cent.: what must be considered the worth of the note on the day of discount?

77. What is the value of $1000 of bank stock at 105 per cent., or 5 per cent. advance?

78. What is the least common multiple of 2, 7, 14 and 49?

79. A merchant shipped to Havana 110bbl. flour, at $5.75 per barrel, and received in return 325 boxes of sugar, at $3.40 per box: what amount does he still owe?

80. The double and the half of a certain number, increased by 2½, make 100: what is the number?

APPENDIX

BOOK-KEEPING BY SINGLE ENTRY

Book-keeping explained.

Book-keeping is a plan adopted by business men to facilitate the interchanges of trade, and is simply an arithmetical record of mercantile transactions. It is of two kinds, single and double entry. The latter is used when commercial engagements are very extensive and great accuracy is desired; but the former is most simple, and is sufficiently methodical for ordinary trade.

Single Entry is the kind treated of in this work.

The day book.

The *Day Book* is for all mercantile charges and credits. At short intervals these should be posted, that is, transferred in date and amount to a book called

The ledger.

The *Ledger*. This contains the summary account of each individual whose name is in the Day Book, with numbers referring to the specific page of transactions.

The cash book.

The *Cash Book* shows receipts and expenditures.

The bank book.

The *Bank Book* shows the deposits and withdrawals of sums lodged in a bank for safety and convenience in transacting business.

The bills payable.

The *Bills and Notes Payable* shows to whom and when an amount is due; and

The bills receivable.

The *Bills and Notes Receivable*, by whom a stated sum is to be paid, and the time of expiration of credit.

REMARKS ON NOTES.

A joint and several note. A joint and several note can be collected by either of the signers.

The endorser. The endorser of note is liable for the amount.

A note not negotiable. A note is not negotiable when the words payable "to order" are omitted.

Notes without value. All notes, without the expression "for value received," are valueless.

When payable to bearer. When a note is payable to A B or bearer, the signer is responsible to the presenter only.

Days of grace. A note payable at a bank has an extension of 3 days beyond its stated term. This is called grace. When the last of these days happens on Sunday or any recognized holiday, payment must be made on the day preceding.

Business notes: Notes passing commercially through banks are either those given in payment of a purchase, and therefore called business notes, or those for which Accommodation paper: the bank has advanced money, called accommodation paper. Notes are often lodged in banks by Collection notes: holders simply for collection. When a note remains unpaid it is protested, or a notification made to par-Notes unpaid. ties interested of its non-payment.

COMMERCIAL FORMS

NEGOTIABLE NOTES.

CHARLESTON, S. C., *Feb.* 4, 1863.

For value received, I promise to pay Messrs McCarter and Dawson, or order, five hundred dollars, on demand, with interest.

WASHINGTON IRVING.

$500.

—

SAVANNAH, GA., *June* 19, 1865.

Four months from date, I promise to pay Oglethorpe Hall, or order, three hundred dollars, for value received.

STEPHEN DRAYTON.

$300.

—

NEW ORLEANS, *April* 5, 1863.

Sixty days from date, we jointly and severally promise to pay Mr. F W Pickens seven hundred dollars, for value received.

JAMES J. McCARTER,
EDMUND DAWSON.

$700.

—

FORM OF ORDER.

MOBILE, *July* 4, 1863.

Messrs. HAYNE, AIKEN & Co.

Gentlemen: Please pay to the order of Hon. C. G. Memminger two thousand dollars, and charge to

Your obedient servant,

JEFFERSON DAVIS.

Note.—The order, before it can be paid, must be endorsed.

RECEIPTS.

COLUMBIA, S. C., *April* 19, 1864.

Received of Ethelwald Preston one hundred dollars, on account.

WILLIAM MANNING.

KNOXVILLE, TENN., *Aug.* 9, 1865.

Received of James Montgomery three hundred dollars, in full of all demands.

JOSEPH ADDINGTON.

$300.

FORM OF A BILL.

RICHMOND, VA., Jan. 1, 1864.

Mr. HENRY ADDINGTON,

Bought of PALMERSTON, RUSSELL & Co.

1 bale Plains, 500 yards, @ 30cts............	$150
2 bales Homespun, 20 pieces, 600 yards, @ 7cts.	42
4 dozen Blankets, @ $3 per pair................	72
4 dozen Scotch caps, @ $4............................	16
	$280

Received Payment,

PALMERSTON, RUSSELL & CO.

DAY BOOK.

REMARKS.				
		January 1, 1862.		
		WILLIAM WORDSWORTH, Dr.		
6m. cr.	E.	To 5 pcs. Calico, 150 yds., @ 10c.	$15 00	
		20 pcs. Shirting, 600 yds., @ 12½c.	75 00	
		5 pcs. Osnaburgs, 100 yds., @ 10c.	10 00	
		10 pcs. Sheeting, 200 yds., @ 25c.	50 00	150 00
		5.		
4m.	E.	JAMES ARGYLE, Dr.		
		To 10 pcs. Scotch Plaid, 400 yds., @ 75c.		300 00
		20.		
Paid to B & C, Factors.	E.	WILLIAM WORDSWORTH, Dr.		
		To Cash paid his order,		55 45
		30.		
		WILLIAM WORDSWORTH, Dr.		
4m.	E.	To 100 pcs. Kersey, 300 yds., @ 50c.	150 00	
		50 pcs. Plains, 100 yds., @ 30c.	30 00	
		50 pcs. Bro. Homespun, 2000 yds, @ 5c.	100 00	
		20 pcs. Chintz, 600 yds., @ 6c.	36 00	
		1 doz. Hdkfs.	1 00	317 00
		19.		
	E.	WILLIAM WORDSWORTH, Dr.		
		To pd. his order to Walter Scott,		19 35
		March 9.		
		JAMES ARGYLE, Dr.		
30 days.	E.	To 2 pcs. Kersey, 100 yds., @ 50c.	50 00	
		20 pcs. Bro. Homespun, 600 yds., @ 5c.	30 00	
		60 pcs. Long Cloth, 1200 yds., @ 10c.	120 00	200 00
		April 6.		
	E.	JAMES ARGYLE, Cr.		
		By Cash,		350 00
		9.		
		JAMES ARGYLE, Dr.		
		To 20 pcs. Calico, 600 yds., @ 8⅓c.		50 00
		July 1.		
	E.	WILLIAM WORDSWORTH, Cr.		
		By Cash,		600 00
		4.		
	E.	*Cash Account*, Cr		
		By Cash pd. travelling expen.		350
		21.		
3m.	E.	WILLIAM WORDSWORTH, Dr.		
		To 2 cases Shirtings, 3000 yds., @ 10c.		300 00

Remarks.						
		July 24, 1862.				
	E.	James Argyle, Cr.				
		By order on Stuart & Co.			275	00
		Aug. 5.				
	E	William Wordsworth, Dr.				
		To 1 case plains, 40 pcs. 1000 yds., @ 50c.			500	00
L'dg'd for		10.				
Collect'n	E.	James Argyle, Cr.				
in Bk. St.		By note at 3m.			137	59
		20.				
4m.	E.	James Argyle, Dr.				
		To 10 pcs. Plains, 300 yds., @ 30c.	90	00		
		2 doz. Hdkfs., @ $2\frac{3}{4}$	5	50	95	50
		Sept. 5.				
	E.	William Wordsworth, Cr.				
		By Cash,			500	00
		Oct. 9.				
	E.	William Wordsworth, Dr.				
		To 1 bale Blankets, 200 pr., @ $2.			400	00
		12.				
		James Argyle, Dr.				
6m.	E.	To 1 case Ga. Plains, 1000 yds., @ 30c.	300	00		
		1 case Bro. Homesp., 1000 yds. @ 6c.	60	00		
		1 case Blankets, 100 pairs, @ $2.	200	00		
		20 pcs. Long Cloth, 400 yds., @ 10c.	40	00	600	00
		Nov. 5.				
	E.	William Wordsworth, Cr.				
		By note at 3m.			600	00
		6.				
	E.	Jas. Argyle, Cr.				
		By Cash,			331	41
		Dec. 1.				
	E.	Wm. Wordsworth, Cr.				
		By bal. to new account,			41	00
		6.				
	E.	Jas. Argyle, Cr.				
		By Cash to bal.			327	00
		Jan. 1, 1863.				
	E.	Wm. Wordsworth, Dr.				
		To bal. from old acct.			41	00
		10.				
	E.	Wm. Wordsworth, Dr.				
		To 2 bales Broadcloth, 300 yds., @ $2.			600	00
	E.	Wm. Wordsworth, Cr.				
		By Cash,			350	00

Note.—The letter E shows that the charge has been entered into the Ledger.

Note.—It will be noticed that a few names and entries only are given. These are sufficient to show the method.

LEDGER.

Dr. WILLIAM WORDSWORTH. Cr.

1862.	Page				1861.	Page			
Jan. 1	1	To am't fr. D.B.	$150	00	July 1	9	By am't fr. D.B.	$600	00
20	2	" "	55	45	Sept. 5	11	" "	500	00
30	6	" "	317	00	Nov. 5	13	"	600	00
Feb. 19	7	" "	19	35	Dec. 1	17	Bal. to new ac't	41	00
July 21	9	" "	300	00					
Aug. 5	10	"	500	00					
Oct. 12	12	" "	400	00					
			1741	00				1741	00
1863.					1863.				
Jan. 1	20	To bal.	41	00	Jan. 10	21	By am't fr. D.B.	250	00
10	21	" am't fr. D.B.	600	00					

Dr. JAMES ARGYLE. Cr.

1862.	Page				1862.	Page			
Jan. 5	8	To am't fr. D.B.	$300	00	April 6	15	By am't fr. D.B.	$350	00
Mar. 9	10	" "	200	00	July 24	20	" "	275	00
April 9	15	" "	50	00	Aug. 10	25	" "	137	59
July 29	25	" "	175	50	Nov. 6	35	" "	331	41
Aug. 20	30	" "	95	50	Dec. 6	62	" "	327	00
Oct. 12	50	" "	600	00					
			1421	00				1421	00

CASH BOOK.

Dr.		CASH.					Cr.
1862.				1862.			
Jan. 1	To Cash on hand	$1000 00		Feb. 1	By pd. note to Bk. S. C.	$550 00	
April 6	To Cash fr. J. Argyle	350 00			By pd. rent	200 00	
July 1	To Cash fr. W. Wordsworth	600 00			By Sund. for house	250 00	
24	To Cash Stuart & Co.	275 00		July 4	By Trav. Exp.	350 00	
Sept. 5	To Cash W. Wordsworth	500 00			Cash Sales	2000 00	
Nov. 6	To Cash J. Argyle	331 41			Bal. to new acc't	38 41	3383 43
Dec. 6	To Cash J. Argyle	327 00	3383 43				
1863.							
Jan. 1	To bal.	38 41					

Note—Though not inserted in our form, yet all receipts and expenditures are to be entered in Day Book.

BANK BOOK.

Dr. BANK OF THE STATE. Cr.

1862.						1862.					
Jan. 1	To deposit	$400	00			Jan. 3	By check to Pringle & Co.	$300	00		
5	" "	600	00			5	By check to Alston & Co.	200	00		
7	" "	300	00			8	By check to Hayne & Co.	400	00		
9	" "	450	00			15	By check to Colcock & Co.	600	00		
10	" "	100	00			16	By check to Coffin & Co.	300	00		
14	" "	700	00			30	By bal.	2717	00		
15	" "	1550	00								
17	" "	149	67								
30	" "	267	33	4517	00			4517	00	4517	00
Feb. 1	To bal.	2717	00								

AMOUNTS—WHEN DUE.

Date.	By whom.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Jan. 3.	B. C.					100																					100					
15.	D. E.						50															40							50			
20.	J. A. & Co.				150																											90
Feb. 1.	A. B.							300																								
5.	G. H.											700																				
May 1.	D. E.													50				50												87		
10.	G. H.	65			210													40														
20.	A. B.												37																			
June 1.	H. J. & Co.																		90													
July 2.	B. D.								65											55		125										55
Aug. 5.	G. H.														65																	
Oct. 9.	N. M.															75																
Nov. 4.	P. R. & Co.									125							87															
Dec. 1.	J. R.										165																					

Note.—This form, it will be noticed, is intended for each month.

BILLS AND NOTES PAYABLE, 1864.

1864.		AMOUNTS—WHEN DUE.																														
Date.	To whom.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Jan. 1.	A. B. & Co.				150																											
9.	B. C.								90																						50	
Feb. 5.	D. E.				35							200					160			100												100
April 6.	E. F									700							90												37			
10.	J. J. & Co.					80																										
	A. B.							90																								
May 1.	D. E.																							415			675		100			
9.	G. H.																	125														
15.	B. C.								150																							
Aug. 1.	L. M.																									325						
7.	N. O.									300										300			500							75		
Oct. 1.	P. Q.	50													50																	
Nov. 2.	E. S.																															
3.	C. D.		75																													
Dec. 5.	A. B.					120																										

Note.—This form, it will be noticed, is intended for each month.

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